Chapter 4 Lines

pages 141–238

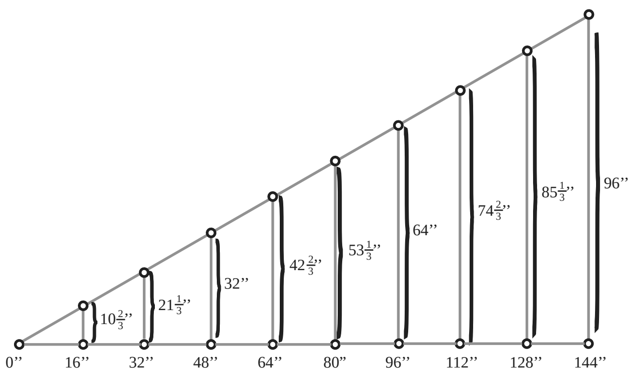
INVESTIGATION 4A ALL ABOUT SLOPE

4.01 Getting Started

For You to Explore

1. The roof in picture (c) is steepest.

2. (a) The roof rises 8 feet over 12 feet of run. You could also say that for every 3 feet of run, it will rise 2 feet. Carpenters would say that the roof has a pitch of 8-12, or an 8-pitch. See diagram below:



(b) There are 9 studs, evenly spaced. 12 feet = 144 inches, so each stud is placed every $\frac{144}{9}$ inches = 16 inches.

(c) The tallest stud is 8 feet = 96 inches. Since the heights of the studs are also evenly distributed, each stud will be $\frac{96}{9}$ inches = $10\frac{2}{3}$ inches taller than the previous stud. The lengths will be $10\frac{2}{3}$ in., $21\frac{1}{3}$ in., 32 in., $42\frac{2}{3}$ in., $53\frac{1}{3}$ in., 64 in., $74\frac{2}{3}$ in., $85\frac{1}{3}$ in., and 96 in. the diagram above.

3. (a) Sasha is cycling faster.

(b) Sasha and Tony have cycled the same distance, so the point where their graphs cross is the point where Sasha catches up with Tony.

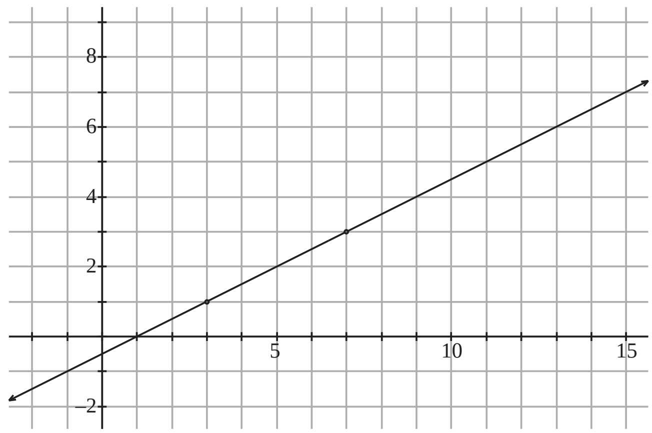
(c) Sasha is cycling at 6 miles per hour.

(d) Tony is cycling at 4 miles per hour.

4. The equations in parts a, c, f, g, and j are all lines.

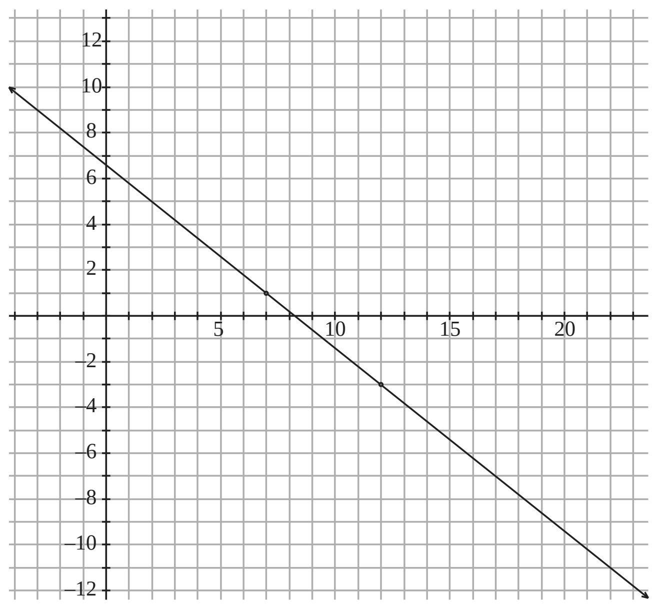
Each equation starts with $y =$ on the left, and y does not appear in the right side of the equation. The distinction between the linear equations and the rest is that the variable x is not raised to any power, nor is it in the denominator or under a radical. Any time a y or an x appears in the equation, it is only multiplied by a constant. All linear equations can be rewritten in the form $ax + by = c$, where a, b, and c are numbers.

5.



Answers may vary. If you are careful when you draw your line, and if you use graph paper, you might be able to see that the line also goes through the points (1, 0), (5, 2), and (9, 4).

6.



Answers may vary. If you are careful when you draw your line, and if you use graph paper, you might be able to see that the line also goes through the points (2, 5), (−3, 9), and (17, −7).

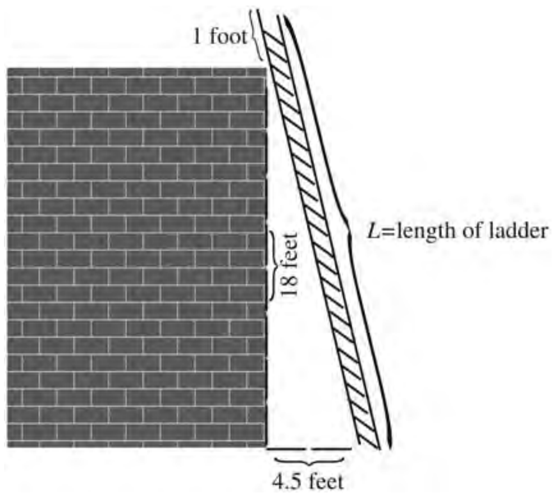
On Your Own

7. (a) The label states that the base should be 1 foot away for every 4 feet of height the top of the ladder reaches. So that means if the top of the ladder is 4 feet up the wall, the base is 1 foot away from the wall. If the top of the ladder is 8 feet up the wall, the base is 2 feet away. And if the top is 12 feet up the wall, the base would have to be 3 feet away. So the answer is 3 feet.

(b) From the answer to the previous part, you know that the base has to be between 2 and 3 feet away. Well, 10 feet is in the exact middle between 8 feet and 12 feet, so the distance away from the wall would be in the exact middle between 2 and 3 feet. So the answer is 2.5 feet.

(c) You may have noticed that if you divide the height by 4, you get the minimum safe distance the base should be from the wall.

(d) You can find out how far away from the wall the base should be by applying the rule you just found. If the height of the wall is 18 feet, the base needs to be $\frac{18}{4}$ feet $= 4.5$ feet from the wall. Now, you have a right triangle, and you know the lengths of the two legs.



Solve for L: Since the ladder extends 1 foot beyond the top of the roof, $L - 1$ will be the hypotenuse of the right triangle with 18 and 4.5 as the legs. Now you can use the Pythagorean Theorem.

$$\begin{aligned} 18^2 + 4.5^2 &= (L-1)^2 \\ 324 + 20.25 &= (L-1)^2 \\ 344.25 &= (L-1)^2 \\ \sqrt{344.25} &= L - 1 \\ 18.55 &\approx L - 1 \\ 19.55 &\approx L \end{aligned}$$

So the ladder should be at least 19.6 feet long.

8. (a) Points in a vertical line all have the same x-coordinate, so an equation would be $x = 5$.

(b) Points in a horizontal line all have the same y-coordinate, so an equation would be $y = -1$.

(c) The point where they intersect has to have x-coordinate 5, and y-coordinate -1. That would be the point $(5, -1)$

9. (a) All three points have the same y-coordinate, so they fall along the same horizontal line, the line $y = 3$.

(b) All three points have the same x-coordinate, so they fall along the same vertical line, the line $x = -3$.

(c) The coordinates are all different, so the points certainly do not fall on the same horizontal or vertical line. If you sketch the three points, it looks like they are on the same line. But how can you be sure? You may have realized from the For You to Explore problems that the ratio of the rise and run for points along the same line are proportional. So the rise from A to B is $15 - 3 = 12$, and the run is $4 - (-1) = 5$. The rise from B to C is $39 - 15 = 24$, and the run is $14 - 4 = 10$.

Since

$$\frac{12}{5} = \frac{24}{10}$$

the points A, B, and C must be on the same line.

(d) Like part (c), the points are not on a horizontal or vertical line. Also, sketching the points, they look like they might be on the same line. Again, to be sure, check the proportion of the rise and the run:

The rise from E to F is $6 - 5 = 1$, and the run is $4 - 3 = 1$.

The rise from F to G is $13 - 6 = 7$, and the run is $10 - 4 = 6$.

Since

$$\frac{1}{1} \neq \frac{7}{6}$$

the points E, F, and G must not be on the same line.

10. If Maria rode 8 miles in 1 hour, her speed was 8 miles per hour.

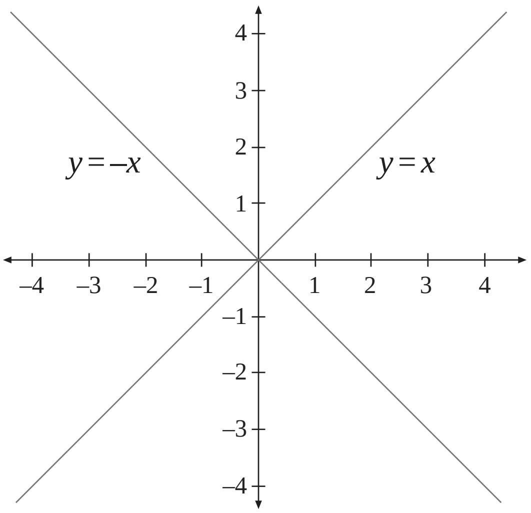
11. (a) To find out how long it took Maria to ride 8 miles, draw a horizontal line from 8 miles on the y-axis to the graph and mark the point on the graph. Then draw a vertical line from the point to the x-axis. That line hits the x-axis at 60, so it took Maria 60 minutes, or 1 hour, to ride 8 miles.

(b) Maria did not travel at a constant speed. The graph is bumpy, so where it is flatter, she was going slower, and where it is steeper, she was going faster.

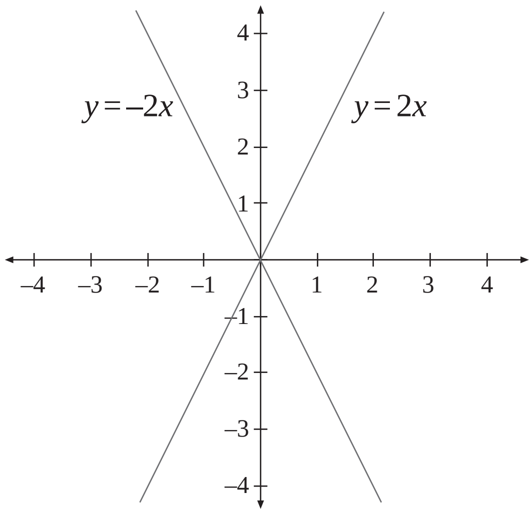
(c) 8 miles per hour is her average speed. If you connected the starting and ending points on the graph, it is the slope of this line.

Maintain Your Skills

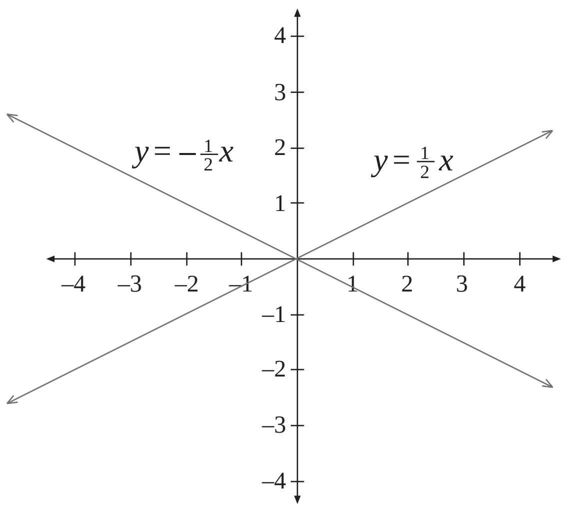
12. (a)



(b)



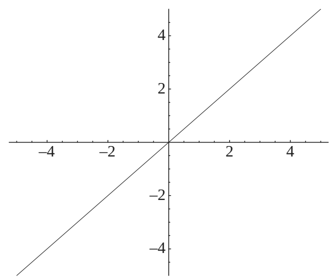
(c)



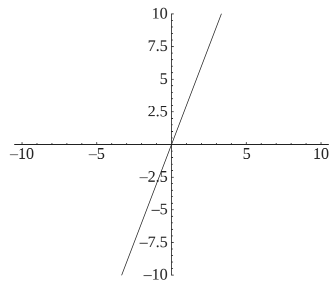
(d) There are a number of patterns you might see:

- If the number in front of the x is positive, the line will rise from the lower left to the upper right. If the number in front of the x is negative, the line will rise from the lower right to the upper left.
- $y = 2x$ is steeper than $y = x$, which is steeper than $y = \frac{1}{2}x$. Likewise, $y = -2x$ is steeper (in the other direction) than $y = x$, which is steeper than $y = -\frac{1}{2}x$.
- $y = 2x$ and $y = -2x$ have the same steepness, only in different directions. The two lines are mirror images of each other, reflected over the y-axis.
- The pairs of lines are also mirror images reflected over the x-axis.

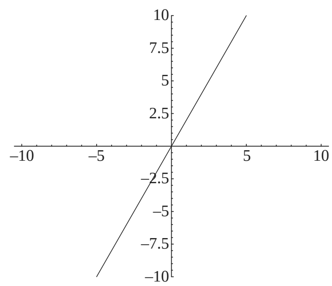
13. (a) $y = x$



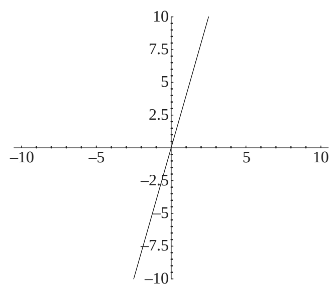
(b) $y = 2x$



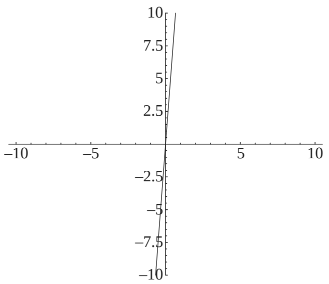
(c) $y = 3x$



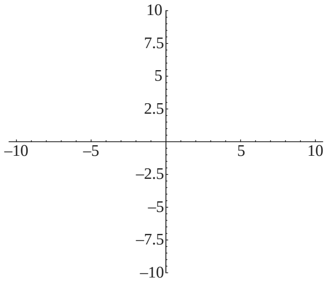
(d) $y = 4x$



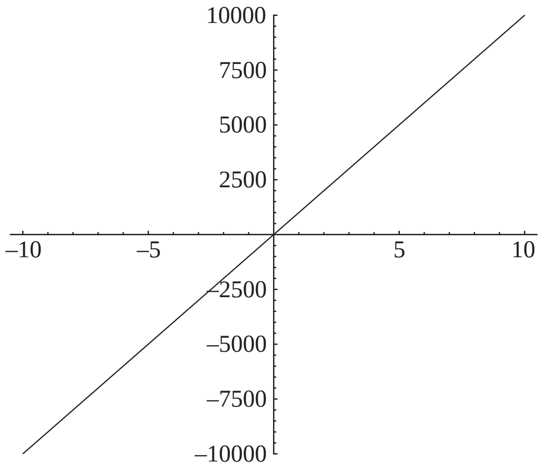
(e) $y = 15x$



(f) $y = 1000x$

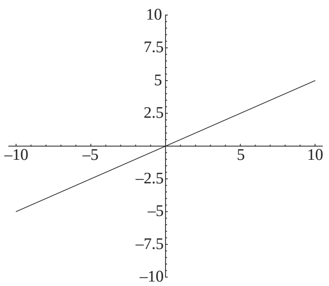


Where is the graph of equation (f)? The graph is so steep that, given the scale of the axes, you cannot distinguish the graph from the y-axis. But, if the scale is changed, the following graph is a result:

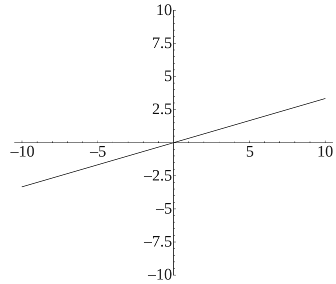


Notice that, as the slope of the line gets larger, the graphs get steeper.

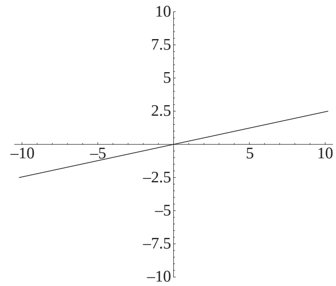
14. (a) $y = \frac{1}{2}x$



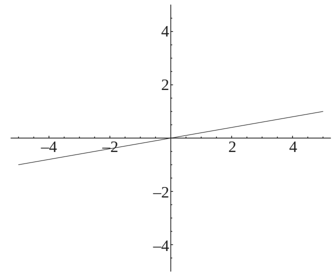
(b) $y = \frac{1}{3}x$



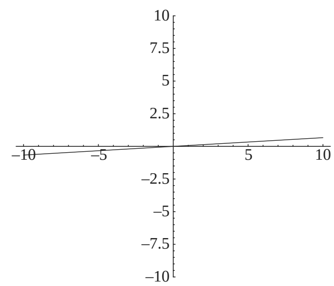
(c) $y = \frac{1}{4}x$



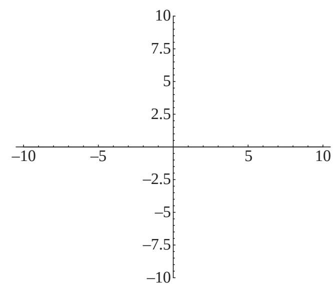
(d) $y = \frac{1}{5}x$



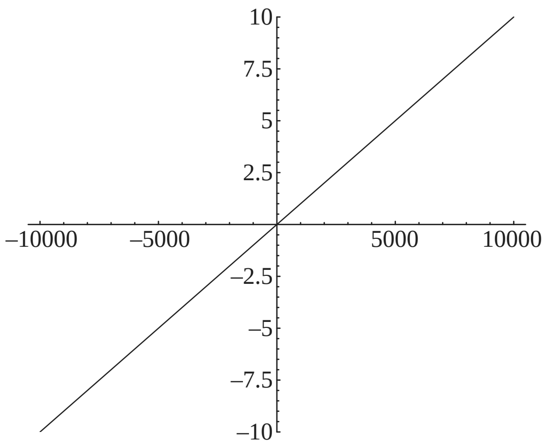
(e) $y = \frac{1}{15}x$



(f) $y = \frac{1}{1000}x$



Where is the graph of equation (f)? The graph is so flat that, given the scale of our axes, you cannot distinguish the graph from the x-axis. But, if the scale is changed, the graph becomes



Notice that, as the slope of the line gets closer to 0, the graphs get flatter.

4.02 Pitch and Slope

Check Your Understanding

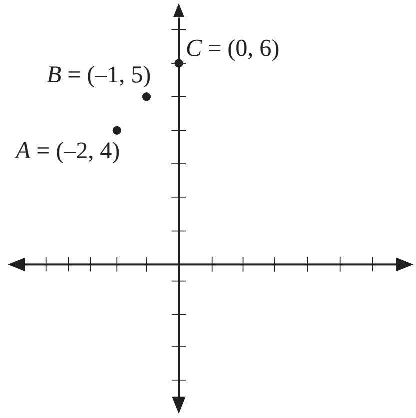
1. To calculate the slope, use the formula

$$m = \frac{\Delta y}{\Delta x}$$

and make sure that when you subtract, you take the coordinates from the two points in the same order.

(a) $m = \frac{8-1}{6-2} = \frac{7}{4}$
(b) $m = \frac{1-8}{2-6} = \frac{-7}{-4} = \frac{7}{4}$
(c) $m = \frac{10-2}{3-12} = \frac{8}{-9} = -\frac{8}{9}$
(d) $m = \frac{10-6}{3-\frac{15}{2}} = \frac{4}{\frac{-9}{2}} = -\frac{8}{9}$
(e) $m = \frac{6-2}{\frac{15}{2}-12} = \frac{4}{\frac{-9}{2}} = -\frac{8}{9}$
(f) $m = \frac{5-0}{-4-0} = \frac{5}{-4} = -\frac{5}{4}$
(g) $m = \frac{4-0}{5-0} = \frac{4}{5}$
(h) $m = \frac{10-0}{-8-0} = \frac{10}{-8} = -\frac{5}{4}$
(i) $m = \frac{5-5}{12-(-4)} = \frac{0}{16} = 0$
(j) $m = \frac{4-4}{5-(-20)} = \frac{0}{25} = 0$
(k) $m = \frac{25-3}{25-3} = \frac{22}{22} = 1$
(l) $m = \frac{5-(-7)}{4-4} = \frac{12}{0}$
You cannot divide by zero, so the slope is undefined.

2. (a)



One possible answer is $C = (0, 6)$.

First, find the slope from A to B.

$$\begin{aligned} m(A, B) &= \frac{\Delta y}{\Delta x} \\ &= \frac{4-5}{(-2)-(-1)} \\ &= \frac{-1}{-1} \\ &= 1 \end{aligned}$$

So $m(A, B) = 1$. To find a C, let C be the point (x, y). Then to find the slope between A and C, use the formula

$$m(A, C) = \frac{\Delta y}{\Delta x} = \frac{y-4}{x-(-2)}$$

Remember that $m(A, C) = m(A, B) = 1$, so you want

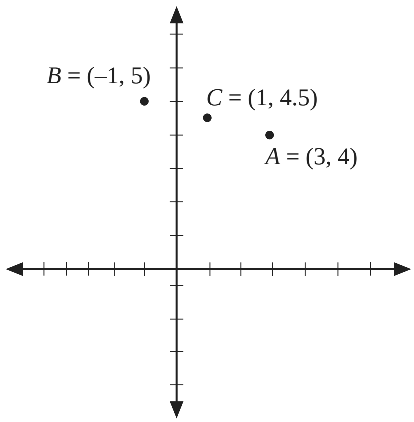
$$\frac{y-4}{x+2} = 1$$

Or, you want $y - 4 = x + 2$. So pick any value for x (other than 3 or -1, since those are the x-coordinates of A and B) and solve for y. You can try 0 for x, and you will get

$$\begin{aligned} \frac{y-4}{x+2} &= 1 \\ \frac{y-4}{(0)+2} &= 1 \\ \frac{y-4}{2} \cdot 2 &= 1 \cdot 2 \\ y - 4 + 4 &= 2 + 4 \\ y &= 6 \end{aligned}$$

So the point C is $(0, 6)$, then $m(A, B) = m(A, C)$.

(b)



One possible answer is $C = (1, 4.5)$.

$$m(A, B) = \frac{4-5}{3-(-1)} = \frac{-1}{4}$$

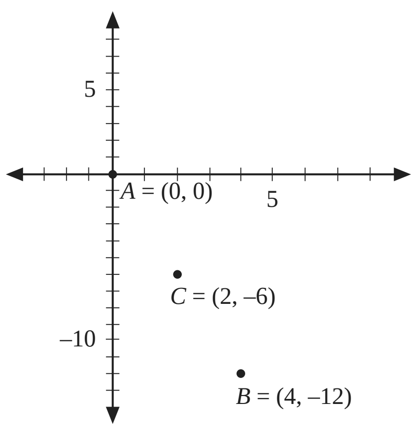
Again, let $C = (x, y)$:

$$m(A, C) = \frac{y-4}{x-3} = -\frac{1}{4}$$

Pick any value for x other than 3 or -1. If you try 1, you get

$$\frac{y-4}{x-3} = -\frac{1}{4}$$
$$\frac{y-4}{(1)-3} = -\frac{1}{4}$$
$$\frac{y-4}{-2} \cdot (-2) = -\frac{1}{4} \cdot (-2)$$
$$y - 4 + 4 = \frac{1}{2} + 4$$
$$y = 4\frac{1}{2}$$

(c)



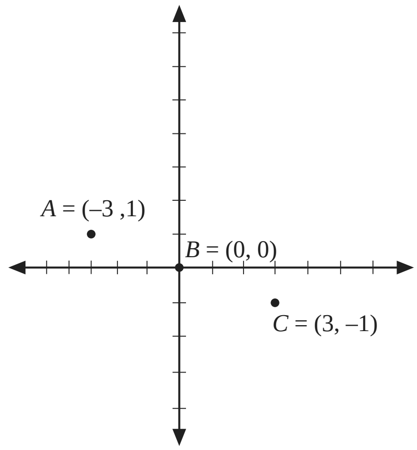
One possible answer is $C = (2, -6)$.

$$m(A, B) = \frac{0-(-12)}{0-4} = \frac{12}{-4} = -3$$

Follow the same procedure from part (a) or (b) to check your C. The equation you should be working with will look like

$$\frac{y-0}{x-0} = -3$$

(d)



One possible answer is $C = (3, -1)$.

$$m(A, B) = \frac{0-1}{0-(-3)} = \frac{-1}{3} = -\frac{1}{3}$$

Follow the same procedure from part (a) or (b) to check your C. The equation you should be working with will look like

$$\frac{y-1}{x+3} = -\frac{1}{3}$$

(e) You can always draw a line that includes any 2 points you pick on the plane. You cannot always draw a single line that includes 3 or more points–more often than not, the 3 points will form a triangle. But, if you measure the slope between the points in pairs, and those slopes are equal, it appears that the points *will* fall on the same line.

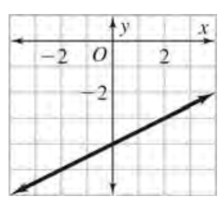
3. (a) Answers will vary. Whichever point has the smaller x-coordinate should also have the *smaller* y-coordinate.

(b) Answers will vary. Whichever point has the smaller x-coordinate should also have the *larger* y-coordinate.

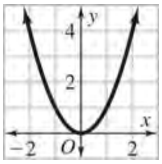
(c) Answers will vary. The points should have the same y-coordinate.

(d) Answers will vary. The points should have the same x-coordinate.

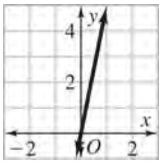
4. (a) Constant slope.



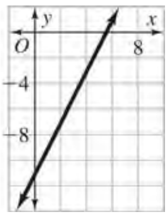
(b) Nonconstant slope.



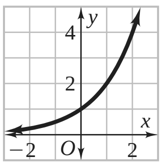
(c) Constant slope.



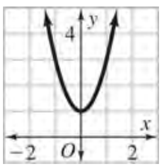
(d) Constant slope.



(e) Nonconstant slope.



(f) Nonconstant slope.



The graphs that have constant slope are straight lines.

5. Answers will vary. Some examples include engineering, mountain climbing, skiing, and architecture.

6. Normally, you would use the two points to calculate the slope:

$$m(J, K) = \frac{r-3}{9-6}$$

So $m(J, K) = \frac{r-3}{3}$. You also know, from the exercise, that $m(J, K) = -\frac{1}{3}$. So, you can say that

$$\frac{r-3}{3} = -\frac{1}{3}$$

and solve for r:

$$\frac{r-3}{3} = -\frac{1}{3}$$
$$\frac{r-3}{3} \cdot 3 = -\frac{1}{3} \cdot 3$$
$$r - 3 = -1$$
$$r - 3 + 3 = -1 + 3$$
$$r = 2$$

On Your Own

7. To calculate the slope, use the formula

$$m = \frac{\Delta y}{\Delta x}$$

and make sure that when you subtract, you take the coordinates from the two points in the same order.

(a) $m = \frac{0-4}{0-(-6)} = \frac{-4}{6} = -\frac{2}{3}$
(b) $m = \frac{0-6}{0-4} = \frac{-6}{-4} = \frac{3}{2}$
(c) $m = \frac{13-13}{-2\frac{2}{3}-\frac{1}{2}} = \frac{0}{-3\frac{1}{6}} = 0$
(d) $m = \frac{5.2-(-7.1)}{27-27} = \frac{12.3}{0}$, which is undefined.
(e) $m = \frac{8-(-1)}{(-1)-(-6)} = \frac{9}{5}$
(f) $m = \frac{17-(-1)}{4-(-6)} = \frac{18}{10} = \frac{9}{5}$
(g) $m = \frac{8-17}{-1-4} = \frac{-9}{-5} = \frac{9}{5}$
(h) $m = \frac{1-4}{5-3} = \frac{-3}{2} = -\frac{3}{2}$
(i) $m = \frac{4-1}{3-5} = \frac{3}{-2} = -\frac{3}{2}$

8. (a) One possible answer is $B = (4, 0)$:

$$m(A, B) = \frac{3-0}{5-4} = \frac{3}{1} = 3$$

(b) One possible answer is $B = (6, 0)$:

$$m(A, B) = \frac{3-6}{5-4} = \frac{-3}{1} = -3$$

(c) One possible answer is $B = (2, 4)$:

$$m(A, B) = \frac{3-4}{5-2} = \frac{-1}{3} = -\frac{1}{3}$$

(d) One possible answer is $B = (2, 3)$:

$$m(A, B) = \frac{3-3}{5-2} = \frac{0}{3} = 0$$

(e) One possible answer is $B = (10, 6)$:

$$m(A, B) = \frac{3-6}{5-10} = \frac{-3}{-5} = \frac{3}{5} \quad \text{and}$$

$$m(O, B) = \frac{0-6}{0-10} = \frac{-6}{-10} = \frac{3}{5}$$

(f) There are two possible answers: $B = (9, 6)$ or $(1, 0)$. If $B = (9, 6)$ then $m(A, B) = \frac{3-6}{5-9} = \frac{-3}{-4} = \frac{3}{4}$. If $B = (1, 0)$ then $m(A, B) = \frac{3-0}{5-1} = \frac{3}{4}$. Both $(9, 6)$ and $(1, 0)$ are five units away from point A (using the Pythagorean Theorem).

9. (a) One point is above and to the right of the other point.
(b) One point is below and to the right of the other point.
(c) The two points are on a horizontal line.
(d) The two points are on a vertical line.

10. (a)

$$\begin{aligned} m(T, J) &= \frac{4-(-8)}{(-2)-7} \\ &= \frac{12}{-9} \\ &= -\frac{4}{3} \end{aligned}$$

(b) There are many other points on $\overleftrightarrow{TJ}$. From the figure, though, it is pretty clear that the point $P = (1, 0)$ is on the graph of $\overleftrightarrow{TJ}$.

If $P = (1, 0)$, then

$$\begin{aligned} m(T, P) &= \frac{4-0}{(-2)-1} \\ &= \frac{4}{-3} \\ &= -\frac{4}{3} \end{aligned}$$

and

$$\begin{aligned} m(P, J) &= \frac{0-(-8)}{1-7} \\ &= \frac{8}{-6} \\ &= -\frac{4}{3} \end{aligned}$$

The two slopes are equal. That must mean something.

11. You can restate Jenna's method using slope by saying,

> The slope from A to B is the same as the slope from B to C, so the points are on a single line.

In Exercise 10, we saw that the slope from T to J was the same as the slope from T to P, so by Jenna's method, the three points do lie on a line.

12. Find the slope between point A and each of the points in the answer choices:

A: $m = \frac{1-5}{10-4} = \frac{-4}{6} = \frac{-2}{3}$
B: $m = \frac{3-5}{7-4} = \frac{-2}{3}$
C: $m = \frac{7-5}{1-4} = \frac{2}{-3}$
D: $m = \frac{8-5}{2-4} = \frac{3}{-2}$

The slope between A and the point in answer choice D is not $\frac{-2}{3}$, so the coordinates in answer choice D cannot be the coordinates of point B. The correct answer is **D**.

13. (a) Since the house is 28 ft wide, the distance from the edge of the house to the center is 14 in. If the pitch is 6-12, for every 12 in. (or 1 ft) of horizontal run, the roof rises by 6 in. The attic has a horizontal run of 14 ft, so the roof rises by

$$6(14) = 84$$

84 in. = 7 ft.

(b) Each step is 8 in. high and there are 5 steps in the picture, so the porch would be 8 in. $\cdot$ 5 = 40 in. off the ground.

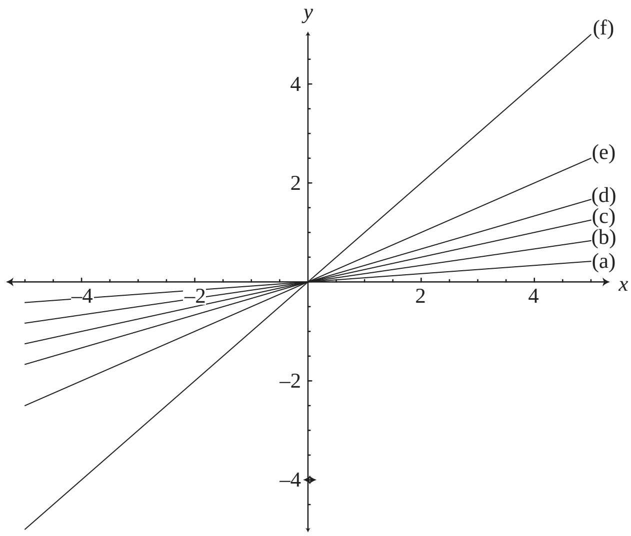
(c) There are 5 steps, but only 4 treads. So the bottom of the house is 4×12 in. $=$ 48 in. from the front of the bottom step.

(d) The attic is 7 ft high. The steps are 40 in. = 3 ft 4 in. It looks like the first floor is a little larger than the attic, about 9 ft. So the house is about 19 ft 4 in. high.

(e) The house is 19 ft 4 in. high, or 232 in. high. From Exercise 7, you know that for each 4 ft of vertical height, the ladder needs to be 1 ft away from the wall. So you would have to place a ladder $\frac{232}{4}$ in. $=$ 58 in., or 4 ft 10 in. from the bottom of the house to reach the peak safely.

Maintain Your Skills

14.



15. (a) You can find the fraction by placing the first number in the numerator, and the second in the denominator, and then reducing the fraction.

$$\begin{array}{ll} 1-12 = \frac{1}{12} & 4-12 = \frac{4}{12} = \frac{1}{3} \\ 2-12 = \frac{2}{12} = \frac{1}{6} & 6-12 = \frac{6}{12} = \frac{1}{2} \\ 3-12 = \frac{3}{12} = \frac{1}{4} & 12-12 = \frac{12}{12} = 1 \end{array}$$

(b) As the fraction becomes greater, the roof gets steeper. In other words, lines with slopes that are closer to zero are closer to the horizontal.

4.03 Rates of Change

Check Your Understanding

1. (a) 75 mph

(b) 0 mph

(c) Answers may vary depending on the estimate of the coordinates, but it's always $m(D, E)$. If $D = (0.5, 37.5)$ and $E = (1.25, 75)$, then

$$m(D, E) = \frac{75-37.5}{1.25-0.5} = \frac{37.5}{0.75} = 50$$

Note that once you pick time coordinates for D and E, you have no choice for the distance coordinates.

2. (a) $200 \cdot 0.03 = 6.00$, so the toll is \$6.
(b)

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{200 \text{ miles}}{3.5 \text{ hours}} \approx 57 \text{ mph}$$

well below the speed limit.

3. (a) The total distance did not change, so the toll is \$6, as in Exercise 2.
(b)

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{200 \text{ miles}}{3 \text{ hours}} \approx 67 \text{ mph}$$

He will more than likely get a speeding ticket.
(c) If the shortest possible time is t, then

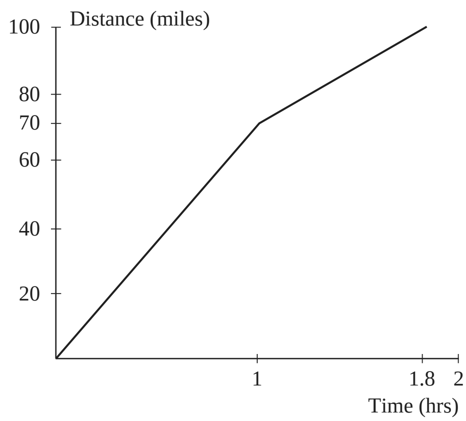
$$\frac{200}{t} = 65$$

so $t \approx$3 hr 5 min, or 3.08 hrs

4. (a) If Kristin averaged 65 mph for the whole trip, it would have taken about 1.8 hrs to travel 120 miles. She has used up 1 of those hours, so the rest of her trip must take at least 0.8 hrs. She has 50 miles to go, so her average speed had better be at most

$$\frac{50}{0.8} = 62.5$$

(b)



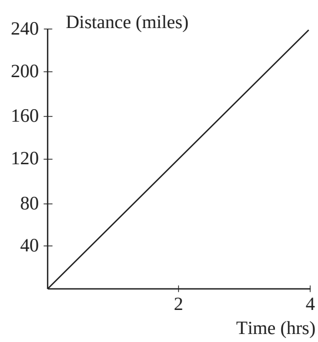
5. (a) Use Guess-Check-Generalize.
Guess #1: Suppose the trip was 350 miles long.
Check: If his ticket showed an average of 70 mph, his total time on the highway (which is also his driving time, if he didn't take a nap) would have been $\frac{350}{70}$, or 5 hours. But if he took a half-hour nap, his driving time would have been 4.5 hours, and his average speed would have been $\frac{350}{4.5}$, or about 78 mph. Close, but not 80 mph.
Guess #2: Suppose the trip was 420 miles long.
Check: If his ticket showed an average of 70 mph, his time without a nap would have been $\frac{420}{70}$, or 6 hours. But if he took a half-hour nap, his time would have been 5.5 hours, and his average speed would have been $\frac{420}{5.5}$, or about 76 mph. Worse than before, but that's OK.
Generalize: The distance was divided by 70, subtracted 0.5, divided that into the distance and checked to see if the answer was 80. So, for any distance d the equation is

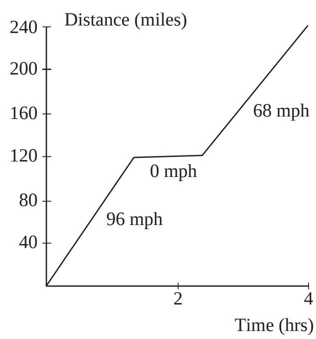
$$\frac{d}{\frac{d}{70} - 0.5} = 80$$

Don't get nervous. Just multiply both sides by $\frac{d}{70} - 0.5$

$$\begin{aligned}
\frac{d}{\frac{d}{70} - 0.5} \cdot \left(\frac{d}{70} - 0.5\right) &= 80 \cdot \left(\frac{d}{70} - 0.5\right) \\
d &= 80\left(\frac{d}{70} - 0.5\right) \\
d &= \frac{8}{7}d - 40 \\
d - \frac{8}{7}d &= \frac{8}{7}d - 40 - \frac{8}{7}d \\
-\frac{1}{7}d &= -40 \\
-\frac{1}{7}d \cdot (-7) &= -40 \cdot (-7) \\
d &= 280
\end{aligned}$$

So Ryota drove 280 miles, and it took him 4 hours.
(b)





On Your Own

6. (a) Car A is the hybrid. Car B is the sedan. Car C is the sport utility vehicle.
(b) The hybrid gets 50 mpg, the sedan gets 30 mpg, and the SUV gets 20 mpg.

7. The equation for transfer rate is

$$\text{transfer rate} = \frac{\Delta\text{amount of data}}{\Delta\text{time}}$$

To figure out the time, let t be the number of seconds. Plug in everything you know and solve for t.

(a)

$$\begin{aligned}1200 &= \frac{49{,}000}{t}\\ 1200 \cdot t &= \frac{49{,}000}{t} \cdot t\\ 1200t &= 49{,}000\\ \frac{1200t}{1200} &= \frac{49{,}000}{1200}\\ t &= \frac{245}{6}\end{aligned}$$

So it takes about 41 seconds to download at 1200 kbps.

(b)

$$\begin{aligned}384 &= \frac{49{,}000}{t}\\ 384t &= 49{,}000\\ t &= \frac{49{,}000}{384} = \frac{6125}{48}\end{aligned}$$

So it takes 128 seconds, or 2 min 8 sec, to download at 384 kbps.

(c) You may have noticed a pattern at this point. In both cases, t ended up being the total size of the file divided by the rate. You can skip ahead to that part to get the answer more quickly.

$$t = \frac{49{,}000}{56} = 875$$

So it takes 875 seconds, or 14 min 35 sec to download at 56 kbps.

(d)

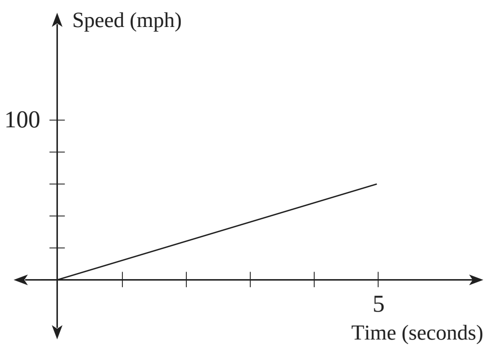
$$t = \frac{49{,}000}{2.4} \approx 20{,}417$$

So it takes about 20,417 seconds, or roughly 5 hours and 40 minutes, to download at 2400 baud.

8. (a) miles per hour

(b) No, nearly any car that isn't badly in need of repair can travel at 60 mph.

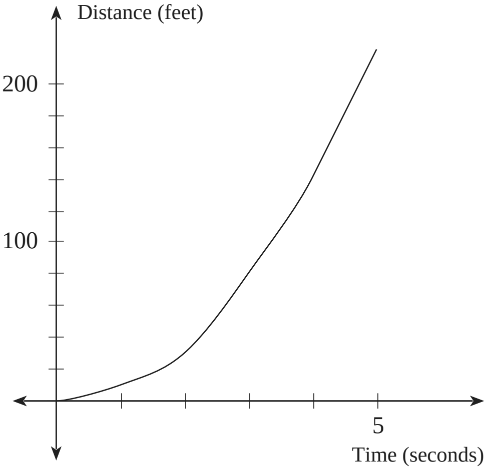
(c)



9. (a) $\frac{60 \text{ mph}}{5 \text{ s}} = 12$ miles per hour per second.

(b) The rate of change of the speed is the acceleration. Powerful cars have high acceleration, rather than high speed. (They may have a higher top speed, too, but any car's top speed is well over the speed limit; it is the acceleration that you notice in actual nonracetrack driving.)

(c)



It is easy to get confused about units of time. The car's final speed is 60 miles per *hour*, but it takes five *seconds* to attain that speed. If you ended up thinking that the car travels 150 miles in five seconds, it's because you used the number 60 as though it were miles per second.

If you did not get exact numbers for the y axis, remember that the main point is to understand that the graph is a curve, not a straight line. But here is how you would get the numbers:

The car accelerates at a rate of

$$\frac{60 \text{ miles per hour}}{5 \text{ seconds}} = 12 \text{ miles per hour per second}$$

but it will be less confusing if time is consistently measured in seconds. Also, in five seconds the car will not travel very far, certainly less than a mile, so a better unit of distance would be feet. First, convert 60 mph to feet per second:

$$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}}$$

Notice that you start with an expression that means "12 miles per hour." The next fraction has a value of 1, since 5280 feet = 1 mile. The last two fractions also equal 1. So, in effect, you are multiplying your original number by a bunch of ones, so the value does not change. Also, you can "cancel the units," meaning the "miles" in the numerator of the first fraction cancels with the "miles" in the denominator of the second, and so forth.

$$\frac{60\,\cancel{\text{miles}}}{1\,\cancel{\text{hour}}} \cdot \frac{5280 \text{ feet}}{1\,\cancel{\text{mile}}} \cdot \frac{1\,\cancel{\text{hour}}}{60\,\cancel{\text{minute}}} \cdot \frac{1\,\cancel{\text{minute}}}{60 \text{ seconds}}$$

And now, you are left with

$$\frac{\cancel{60}^{1} \cdot 5280 \cdot 1 \cdot 1 \text{ feet}}{1 \cdot 1 \cdot \cancel{60}^{1} \cdot 60 \text{ s}} = \frac{5280 \text{ feet}}{60 \text{ s}} = 88 \text{ ft/s}$$

So the car accelerates at a rate of

$$\frac{88 \text{ ft/s}}{5 \text{ s}} = 17.6 \text{ ft/s}^2$$

At the end of the first second, the car is going 17.6 feet per second. Its *average* speed during that second is half that much, 8.8 feet per second. So it travels 8.8 feet.

During the second second, the car accelerates from 17.6 ft/s to 35.2 ft/s. Its average speed during that time is 26.4 ft/s, so it adds 26.4 feet to its distance, for a total of 35.2 feet.

Similar calculations show that the distances at the ends of the third through fifth seconds are 79.2, 140.8, and 220 feet.

It is assumed that the car accelerates at a constant rate during the entire five seconds.

10. (a) She bikes 230 yards per minute, so after 5 minutes she will have biked $5(230 \text{ yards}) = 1150$ yards.

(b) Every minute she bikes 230 yards. So, if she bikes for t minutes, she will go $230t$ yards. So, find t such that

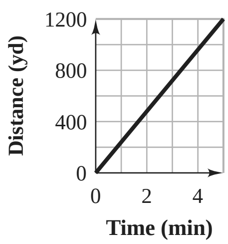
$$230t = 100$$

and solve the equation using the basic moves:

$$230t = 100$$
$$t = \frac{100}{230} = \frac{10}{23}$$

It will take her $\frac{10}{23}$ of a minute to bike 100 yards. $\frac{10}{23}$ of a minute is ≈ 26 seconds.

(c)



Since Katie bikes 230 yards every minute, if d is the distance she has biked, and t is the number of minutes she has been riding, then the following equation results:

$$d = 230t$$

The above graph is the graph of the equation, where the horizontal axis is the t-axis, and the vertical axis is the d-axis.

(d) Choose two easy points like $(0, 0)$ and $(1, 230)$ Both satisfy the equation $d = 230t$, which means that the points are on the graph. The slope between $(0, 0)$ and $(1, 230)$ is

$$\frac{230 - 0}{1 - 0} = \frac{230}{1} = 230$$

Since the graph is a line, the slope between any two points will be the same. So, no matter which two points you choose, you'll get 230.

11. (a) No, Nick cannot catch Katie. Since Katie bikes at a faster rate, Nick's slower biking cannot make up the difference. Katie will get farther and farther ahead.

(b) Yes, Lance will catch Katie. Lance bikes faster than Katie, so he will catch her. He shortens the gap by 70 yards every minute, and has 230 yards to make up, so it will take $\frac{230}{70} \approx 3.29$ minutes to catch Katie.

12. The first mile that Tamara rides takes $\frac{1}{60}$ hours $=$ 1 minute. The second mile takes $\frac{1}{30}$ hours $= 2$ minutes. So, the whole trip takes 3 minutes. Tamara travels 2 miles in 3 minutes, or $\frac{3}{60} = \frac{1}{20}$ hours. The average speed, in miles per hour, is $\frac{2 \text{ mi}}{\frac{1}{20} \text{ h}} = 40$ mi/h. The correct answer is **C**.

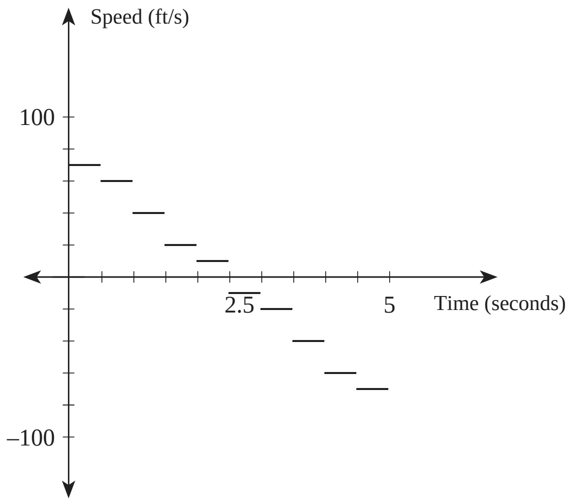
13. (a) Between O and P, then between Q and S, and then between S and U. Constant rate means constant slope means a graph which is a line.

(b) Between Q and S. Zero rate means zero slope.

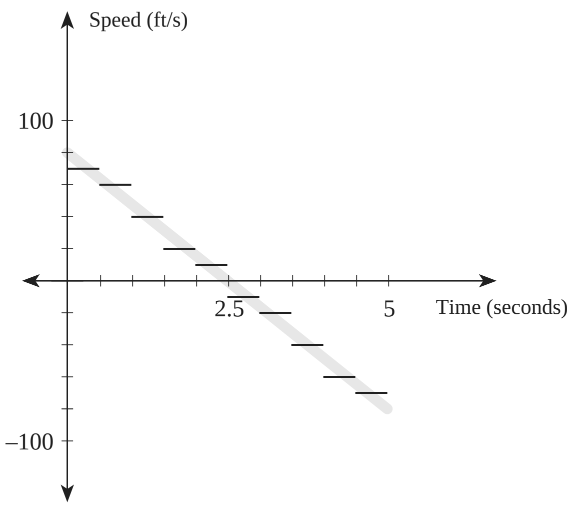
14. Without knowing the exact equation of the graph, the question is difficult to answer. You might estimate that the following points are on the graph:

$(0, 0)$, $(0.5, 35)$, $(1, 65)$, $(1.5, 85)$, $(2, 95)$, $(2.5, 100)$, $(3, 95)$, $(3.5, 85)$, $(4, 65)$, $(4.5, 35)$, $(5, 0)$

Then the rate of change between 0 and 0.5 is 70; between 0.5 and 1 it is 60; between 1 and 1.5 it is 40; and so on. Here is what you get by plotting these approximations:

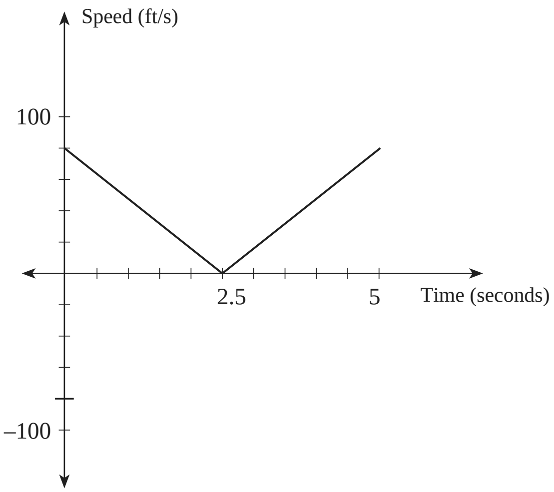


The plot suggests that the actual speed graph is a descending line:



Although you might have some doubt, because the horizontal segments in the picture aren't evenly spaced,

the irregularity comes from estimation error; the actual points on the graph are
(0, 0), (0.5, 36), (1, 64), (1.5, 84), (2, 96), (2.5, 100), (3, 96), (3.5, 84), (4, 64), (4.5, 36), (5, 0)
and the actual speed graph is indeed a line:



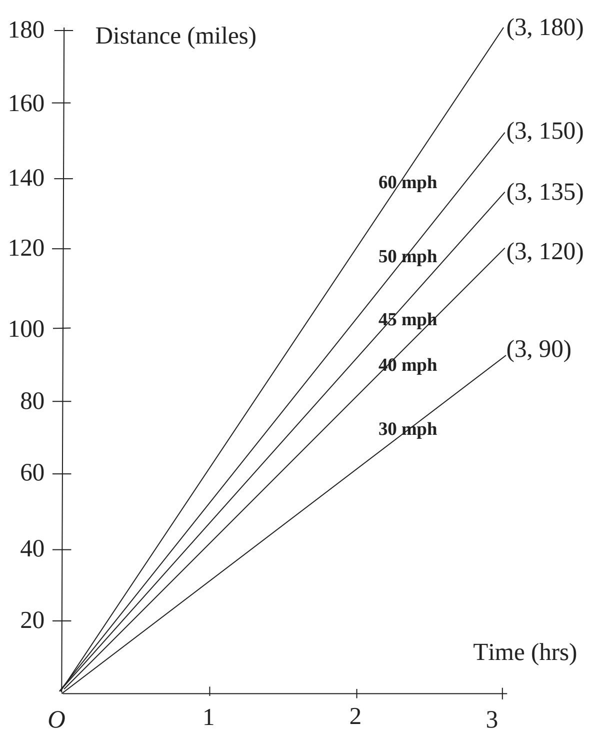
Note that the "flat" point in the middle of the distance graph corresponds to a speed of zero. The speed is positive as the height increases, and negative as the height decreases.

By the way, the position graph has the equation $y = 80x - 16x^2$; the speed graph is $y = 80 - 32x$.

15. Suppose you start at A with a speed of 0, speed up, drive 10 miles in a straight line to B, and then stop. The car's readings at A and B are 0, so the average is 0. But your actual average speed is more than 0, so Kwata is wrong.

Maintain Your Skills

16.



4.04 Collinearity

Check Your Understanding

1. (a)

$$m(A, B) = \frac{15 - 3}{4 - (-1)} = \frac{12}{5} \qquad m(A, C) = \frac{39 - 3}{14 - (-1)} = \frac{36}{15} = \frac{12}{5}$$

Since $m(A, B) = \frac{12}{5} = m(A, C)$, A, B, and C are collinear.

(b)

$$m(X, Y) = \frac{3 - 3}{4 - (-3)} = \frac{0}{7} = 0 \qquad m(X, Z) = \frac{3 - 3}{100 - (-3)} = \frac{0}{103} = 0$$

Since $m(X, Y) = 0 = m(X, Z)$, X, Y, and Z are collinear.

(c)

$$m(P, D) = \frac{2 - 8}{0 - (-3)} = \frac{-6}{3} = -2 \qquad m(P, Q) = \frac{(-4) - 8}{3 - (-3)} = \frac{-12}{6} = -2$$

Since $m(P, D) = -2 = m(X, Z)$, P, D, and Q are collinear.

(d)

$$m(N, M) = \frac{6 - 5}{4 - 3} = \frac{1}{1} = 1 \qquad m(N, D) = \frac{13 - 5}{10 - 3} = \frac{8}{7} = \frac{8}{7}$$

Since $m(N, M) \neq m(N, D)$, N, M, and D are *not* collinear.

2.

$$m(A, B) = \frac{2 - (-1)}{(-1) - (-4)} = \frac{3}{3} = 1 \qquad m(A, C) = \frac{4 - (-1)}{2 - (-4)} = \frac{5}{6} = \frac{5}{6}$$

$$m(A, D) = \frac{6 - (-1)}{3 - (-4)} = \frac{7}{7} = 1$$

Since $m(A, B) = m(A, D)$, A, B, and D are collinear. Since $m(A, C) \neq m(A, B)$, C is not collinear with the other points.

3. There are three ways of taking pairs of points: A and B, B and C, and A and C, which gives you six different possible ways to measure the slopes: $m(A, B)$, $m(A, C)$,

$m(B, A)$, $m(B, C)$, $m(C, A)$, and $m(C, B)$. If the three points are collinear, then these six slopes must be equal. Derman's question does bring up a couple of questions:

(a) Is it always true that $m(A, B) = m(B, A)$?
(b) Why do you only have to compare two of the three slopes? Can you be sure the third one is also equal?

First, show Derman that $m(A, B) = m(B, A)$.
Proof. Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$. Then you can say that

$$\begin{aligned}
m(A, B) &= \frac{(y_2 - y_1)}{(x_2 - x_1)} && \text{def. of slope}\\
&= \frac{-1(y_2 - y_1)}{-1(x_2 - x_1)} && \text{Multiply by } \tfrac{-1}{-1}.\\
&= \frac{-y_2 - (-y_1)}{-x_2 - (-x_1)} && \text{distributive law}\\
&= \frac{-y_2 + y_1}{-x_2 + x_1} && \text{def. of subtraction}\\
&= \frac{y_1 - y_2}{x_1 - x_2} && \text{commutative property}\\
&= m(B, A) && \text{def. of slope}
\end{aligned}$$

So the statement $m(A, B) = m(B, C)$ is always true. Now you know you only have three distinct slopes: $m(A, B)$, $m(A, C)$, and $m(B, C)$.

The assumption on page 343 states that it is sufficient to compare just two of the three slopes to test collinearity. This proof shows that if you know two of the slopes are equal, then the third must also be equal.
Proof. Let $A = (x_a, y_a)$, $B = (x_b, y_b)$, and $C = (x_c, y_c)$. Suppose $m(A, B) = m(B, C) = k$. Then you need to show that $m(A, C) = k$.

$$\begin{aligned}
m(A, B) &= k\\
\frac{y_b - y_a}{x_b - x_a} &= k && \text{def. of slope}\\
y_b - y_a &= k(x_b - x_a) && \text{Multiply both sides by } (x_b - x_a).\\
y_b - y_a &= kx_b - kx_a && \text{distributive law}
\end{aligned}$$

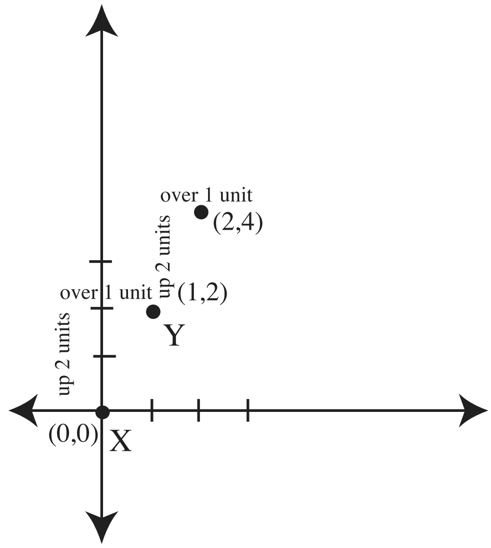
Follow the same steps to get $y_c - y_b = kx_c - kx_b$. You can combine the two equations like this:

$$\begin{array}{r}
y_b - y_a = \qquad kx_b - kx_a\\
+(y_c - y_b) = +(kx_c - kx_b)\\
\hline
y_b - y_a + y_c - y_b = kx_b - kx_a + kx_c - kx_b
\end{array}$$

Notice on the left side, you have y_b and $-y_b$, which combine to get 0. Also, on the right side, you have kx_b and $-kx_b$, which also combine to get 0.

$$\begin{aligned}
-y_a + y_c &= -kx_a + kx_c\\
y_c - y_a &= kx_c - kx_a && \text{commutative property}\\
y_c - y_a &= k(x_c - x_a) && \text{distributive law}\\
\frac{y_c - y_a}{x_c - x_a} &= \frac{k(x_c - x_a)}{x_c - x_a}\\
\frac{y_c - y_a}{x_c - x_a} &= k && \text{canceling}\\
m(A, C) &= k && \text{def. of slope}
\end{aligned}$$

4. (a) X and Y lie on a horizontal line, so any point with a y-coordinate of -4 will lie on $\overleftrightarrow{XY}$.
(b) X and Y lie on a vertical line, so any point with an x-coordinate of -4 will lie on $\overleftrightarrow{XY}$.
(c) The slope between X and Y is 2. Any point whose slope to $(0, 0)$ or $(1, 2)$ is 2 will be on the line. For example, here is one way to get more points on the line:



By continuing in the same direction, points $(2, 4)$ and $(3, 6)$ can be found, and many others. In general, any point with the coordinates $(a, 2a)$ is on the line (where a can be any number).

5. A good point-tester would be to see if the slope between any point and $(2, 3)$ (or $(12, -4)$) is $-\frac{7}{10}$.
An equation point-tester for the point $P = (x, y)$ would be that P is on the line if (x, y) satisfies the following equation:

$$\frac{y-3}{x-2} = -\frac{7}{10}$$

On Your Own

6. Answers will vary. Samples are given.

(a) $A = (0, -3)$ $B = (2, 5)$ $C = (4, 13)$

$$m(A, B) = \frac{-3-5}{0-2} = \frac{-8}{-2} = 4$$
$$m(A, C) = \frac{-3-13}{0-4} = \frac{-16}{-4} = 4$$

So, $m(A, B) = m(A, C)$.

(b) $D = (2, -1)$ $E = (4, -4)$ $F = (8, -10)$

$$m(D, E) = \frac{-1-(-4)}{2-4} = \frac{3}{-2} = -\frac{3}{2}$$
$$m(D, F) = \frac{-1-(-10)}{2-8} = \frac{9}{-6} = -\frac{3}{2}$$

So, $m(D, E) = m(D, F)$.

(c) $G = (0, 0)$ $H = (1, 1)$ $I = (3, 9)$

$$m(G, H) = \frac{0-1}{0-1} = \frac{-1}{-1} = 1$$
$$m(G, I) = \frac{0-9}{0-3} = \frac{-9}{-3} = 2$$

$m(G, H) \neq m(G, I)$ so G, H, and I are not collinear.

(d) $J = (2, -1)$ $K = (4, -4)$ $L = (8, -10)$

$$m(J, K) = \frac{-1-(-4)}{2-4} = \frac{3}{-2} = -\frac{3}{2}$$
$$m(J, L) = \frac{-1-(-10)}{2-8} = \frac{9}{-6} = -\frac{3}{2}$$

$m(J, K) = m(J, L)$, so J, K, and L are collinear.

(e) $M = (0, 2)$ $N = (1, 3)$ $O = (2, 5)$

$$m(M, N) = \frac{2-3}{0-1} = \frac{-1}{-1} = 1$$
$$m(M, O) = \frac{2-5}{0-2} = \frac{-3}{-2} = \frac{3}{2}$$

$m(M, N) \neq m(M, O)$ so M, N, and O are not collinear.

7. The conversion formula is $C = \frac{5}{9}(F - 32)$. To convert $F = -10$:

$$\begin{aligned} C &= \frac{5}{9}(-10-32) \\ &= \frac{5}{9}(-42) \\ &= -23\frac{1}{3} \end{aligned}$$

The answer is **C**.

8. To test whether a point is on ℓ, find the slope between R and S, and test whether the slope between the tested point and R (or S) is the same.
First, find the slope between R and S.

$$m(R, S) = \frac{4-2}{-2-6} = -\frac{1}{4}$$

So, each of these points will be on line ℓ if the slope between the point and R (or S) is $-\frac{1}{4}$.

(a) The slope between $R = (-2, 4)$ and $(1, 3)$ is

$$\begin{aligned} \frac{4-3}{-2-1} &= \frac{1}{-3} \\ &= -\frac{1}{3} \end{aligned}$$

So $(1, 3)$ is *not* on ℓ.

(b) The slope between $R = (-2, 4)$ and $\left(1, \frac{13}{4}\right)$ is

$$\begin{aligned} \frac{4-\frac{13}{4}}{-2-1} &= \frac{\left(\frac{3}{4}\right)}{-3} \\ &= -\frac{1}{4} \end{aligned}$$

So $\left(1, \frac{13}{4}\right)$ is on ℓ.

(c) The slope between $R = (-2, 4)$ and $\left(1, \frac{14}{3}\right)$ is

$$\begin{aligned} \frac{4-\frac{14}{3}}{-2-1} &= \frac{\left(-\frac{1}{3}\right)}{-3} \\ &= \frac{1}{9} \end{aligned}$$

So $\left(1, \frac{14}{3}\right)$ is *not* on ℓ.

(d) The slope between $R = (-2, 4)$ and $\left(2, \frac{13}{4}\right)$ is

$$\begin{aligned} \frac{4-\frac{13}{4}}{-2-2} &= \frac{\left(\frac{3}{4}\right)}{-4} \\ &= -\frac{3}{16} \end{aligned}$$

So $\left(2, \frac{13}{4}\right)$ is *not* on ℓ.

(e) The slope between $R = (-2, 4)$ and $(2, 3)$ is

$$\begin{aligned} \frac{4-3}{-2-2} &= \frac{1}{-4} \\ &= -\frac{1}{4} \end{aligned}$$

So $(2, 3)$ is on ℓ.

(f) The slope between $R = (-2, 4)$ and $\left(2, \frac{14}{3}\right)$ is

$$\begin{aligned} \frac{4-\frac{14}{3}}{-2-2} &= \frac{\left(-\frac{2}{3}\right)}{-4} \\ &= \frac{1}{6} \end{aligned}$$

So $\left(2, \frac{14}{3}\right)$ is *not* on ℓ.

(g) The slope between $R = (-2, 4)$ and $\left(4, \frac{13}{4}\right)$ is

$$\begin{aligned} \frac{4-\frac{13}{4}}{-2-4} &= \frac{-\left(\frac{1}{4}\right)}{-6} \\ &= \frac{1}{24} \end{aligned}$$

So $\left(4, \frac{13}{4}\right)$ is *not* on ℓ.

(h) The slope between $R = (-2, 4)$ and $(2, 2.6)$ is

$$\begin{aligned} \frac{4-2.6}{-2-4} &= \frac{1.4}{-6} \\ &= -\frac{7}{30} \end{aligned}$$

So $(4, 2.6)$ is *not* on ℓ.

(i) The slope between $R = (-2, 4)$ and $\left(4, \frac{5}{2}\right)$ is

$$\begin{aligned} \frac{4-\frac{5}{2}}{-2-4} &= \frac{\left(\frac{3}{2}\right)}{-6} \\ &= -\frac{3}{12} \\ &= -\frac{1}{4} \end{aligned}$$

So $\left(4, \frac{5}{2}\right)$ is on ℓ.

9. You are told that each point is on the line ℓ. You know that in order for the point to be collinear with R and S, the slope between the point and R must equal the slope between R and S, which you know from Exercise 8 is $-\frac{1}{4}$.
So, for each point, test the slope from that point to R, and say it is equal to $-\frac{1}{4}$. Then, solve for whatever variable you have in the point.

(a)

$$\begin{aligned}\frac{4-a}{-2-3}&=-\frac{1}{4}\\ \frac{4-a}{-5}&=-\frac{1}{4}\\ 4-a&=-\frac{1}{4}(-5)\\ 4-a&=\frac{5}{4}\\ -a&=\frac{5}{4}-4\\ a&=\frac{11}{4}\end{aligned}$$

(b)

$$\begin{aligned}\frac{4-b}{-2-5}&=-\frac{1}{4}\\ \frac{4-b}{-7}&=-\frac{1}{4}\\ 4-b&=-\frac{1}{4}(-7)\\ 4-b&=\frac{7}{4}\\ -b&=\frac{7}{4}-4\\ b&=\frac{9}{4}\end{aligned}$$

(c)

$$\begin{aligned}\frac{-4-6}{-2-c}&=-\frac{1}{4}\\ \frac{-10}{-2-c}&=-\frac{1}{4}\\ -10&=-\frac{1}{4}(-2-c)\\ 40&=-2-c\\ c&=-2-40\\ c&=-42\end{aligned}$$

(d)

$$\begin{aligned}\frac{4-d}{-2-7}&=-\frac{1}{4}\\ \frac{4-d}{-9}&=-\frac{1}{4}\\ 4-d&=-\frac{1}{4}(-9)\\ 4-d&=\frac{9}{4}\\ -d&=\frac{9}{4}-4\\ d&=\frac{7}{4}\end{aligned}$$

(e)

$$\begin{aligned}\frac{4-e}{-2-13}&=-\frac{1}{4}\\ \frac{4-e}{-15}&=-\frac{1}{4}\\ 4-e&=-\frac{1}{4}(-15)\\ 4-e&=\frac{15}{4}\\ -e&=\frac{15}{4}-4\\ e&=\frac{1}{4}\end{aligned}$$

(f)

$$\begin{aligned}\frac{4-f}{-2-120}&=-\frac{1}{4}\\ \frac{4-f}{-122}&=-\frac{1}{4}\\ 4-f&=-\frac{1}{4}(-122)\\ 4-f&=\frac{122}{4}\\ -f&=\frac{122}{4}-4\\ -f&=\frac{106}{4}\\ f&=-\frac{53}{2}\end{aligned}$$

(g)

$$\begin{aligned}\frac{4-(-2)}{-2-x}&=-\frac{1}{4}\\ \frac{6}{-2-x}&=-\frac{1}{4}\\ 6&=-\frac{1}{4}(-2-x)\\ 6&=\frac{1}{2}+\frac{x}{4}\\ 6-\frac{1}{2}&=\frac{x}{4}\\ \frac{11}{2}&=\frac{x}{4}\\ 22&=x\end{aligned}$$

(h)

$$\begin{aligned}\frac{4-(-5)}{-2-y}&=-\frac{1}{4}\\ \frac{9}{-2-y}&=-\frac{1}{4}\\ 9&=-\frac{1}{4}(-2-y)\\ 9(4)&=2+y\\ 34&=y\end{aligned}$$

(i)

$$\begin{aligned}\frac{4-12}{-2-z}&=-\frac{1}{4}\\ \frac{-8}{-2-z}&=-\frac{1}{4}\\ -8&=-\frac{1}{4}(-2-z)\\ -8(4)&=(2+z)\\ -32&=2+z\\ -34&=z\end{aligned}$$

(j)
$$\begin{aligned}\frac{4-(v+1)}{-2-v} &= -\frac{1}{4}\\ \frac{3-v}{-2-v} &= -\frac{1}{4}\\ (3-v)(-4) &= (-2-v)\\ 4v-12 &= -2-v\\ 5v &= 10\\ v &= 2\end{aligned}$$

(k)
$$\begin{aligned}\frac{4-(1.5w)}{-2-w} &= -\frac{1}{4}\\ (4-1.5w) &= -\frac{1}{4}(-2-w)\\ (4-1.5w)(-4) &= (-2-w)\\ 6w-16 &= -2-w\\ 7w &= 14\\ w &= 2\end{aligned}$$

(l)
$$\begin{aligned}\frac{4-(u+14)}{-2-u} &= -\frac{1}{4}\\ (-10-u) &= -\frac{1}{4}(-2-u)\\ (-10-u)(-4) &= (-2-u)\\ 40+4u &= -2-u\\ 5u &= -42\\ u &= -\frac{42}{5}\end{aligned}$$

10. To see if a point (x, y) is on the line ℓ, you follow the same checking steps as in the previous exercise. If the slope between (x, y) and R (or S) is $-\frac{1}{4}$ then (x, y) is on line ℓ. So, the equation would be

$$\frac{4-y}{-2-x} = -\frac{1}{4}$$

If (x, y) satisfies the equation, then (x, y) is on line ℓ. The equation can be simplified:

$$\begin{aligned}\frac{4-y}{-2-x} &= -\frac{1}{4}\\ 4(4-y) &= -(-2-x)\\ 16-4y &= 2+x\\ 16-4y+4y &= 2+x+4y\\ 16 &= 2+x+4y\\ 14 &= x+4y\end{aligned}$$

11. The best way to test the equations is to plug in points and make sure each point satisfies the correct answer. You can speed up your work a little, however, by thinking about the points and the equations.

The first point satisfies **A**. You might notice that in equation **A**, the x and y are only multiplied by 1. So this equation does not work for the three points that have a fraction in the y-coordinate, since there's no way you can add a fraction to a whole number and get a whole number. So **A** is out.

For equation **C**, the coefficients of x and y are both positive, but the number on the right side is negative. So the last three points cannot fit, because both the x- and y-coordinates are positive, and there's no way that plugging in two positive numbers on the left side will give you a negative number.

You can also use the same reasoning for equation **D**: since the last three points have positive x- and y-coordinates, there's no way you can get zero by multiplying and adding all positive numbers.

That leaves just **B** as a possible answer. The coefficients are not 1: therefore fractions could go away on the left side, so **B** doesn't have the same problem as **A**. Also, one of the coefficients is negative, so **B** doesn't have the same problem as **C** or **D**. You can double-check each point by plugging it in, and you'll find that they all satisfy the equation, so indeed **B** is the correct answer.

12. (a) To calculate the slope, divide the change in x by the change in y. Starting at the left corner of the house, for every 12 in. horizontal to the right, the house rises by 6 in., so the slope is

$$\frac{6}{12} = \frac{1}{2}$$

(b) Again, divide the change in x by the change in y. Starting at the peak of the house, for every 12 in. horizontal to the right, the house falls by 6 in., so the slope is

$$\frac{-6}{12} = -\frac{1}{2}$$

Did you expect the answers to be opposites?

13. (a) The flatter the roof, the closer to zero the slope will be. On the left side of the carriage house, the slope is always positive, so the flattest part of the roof, in the left corner, will have the smallest slope.

(b) On the left side of the carriage house, the slope is always positive, so the steepest part of the roof, near the top, will have the largest slope.

Maintain Your Skills

14. (a) First, find the slope between two of the first three points. Calculate $m(A, B)$:

$$m(A, B) = \frac{0-0}{7-2} = \frac{0}{5} = 0$$

Next, check if $m(A, X) \stackrel{?}{=} 0$.

$$m(A, X) = \frac{0-0}{7-\frac{14}{9}} = 0$$

Since $m(A, X) = m(A, B)$, X is collinear with A, B, and C.

(b)

$$\begin{aligned}m(A, B) &= \frac{3-3}{3-(-8)} = \frac{0}{11} = 0\\ m(A, X) &= \frac{3-(-3)}{3-(-2)} = \frac{6}{5} \neq 0\end{aligned}$$

Since $m(A, X) \neq m(A, B)$, X is not collinear with A, B, and C.

(c)
$$m(A, B) = \frac{(-2) - (-2)}{5 - \frac{3}{7}} = \frac{0}{\frac{32}{7}} = 0$$
$$m(A, X) = \frac{(-2) - 3}{5 - 2} = \frac{-5}{3} \neq 0$$

Since $m(A, X) \neq m(A, B)$, X is not collinear with A, B, and C.

(d)
$$m(A, B) = \frac{\frac{14}{9} - \frac{14}{9}}{17 - 21} = 0$$
$$m(A, X) = \frac{\frac{14}{9} - \frac{14}{9}}{17 - \frac{17}{9}} = 0$$

Since $m(A, X) = m(A, B)$, X is not collinear with A, B, and C.

(e) In all four parts, A, B, and C all lie on a horizontal line. The previous chapter stated that all points on a horizontal line have the same y-coordinate. So, X is collinear with A, B, and C only if it has the same y-coordinate as A, B, and C.

15. (a) Try calculating $m(A, B)$:
$$m(A, B) = \frac{3 - 9}{4 - 4} = \frac{-6}{0}$$

Oh no, you cannot divide by 0! Is there a way to check if X is collinear with the other three points? Well, if you test $m(B, C)$, you will see that it is also undefined. In fact "undefined" is what you call the slope between two points that are on a vertical line (plot the three points and see that they are indeed on a vertical line). So, if the slope between X and A is also undefined, then it must be collinear with A, B, and C.
$$m(A, X) = \frac{3 - 0}{4 - 4} = \frac{3}{0}$$
so $m(A, X)$ is also undefined.

You may also realize that, like Exercise 14, you do not need to test the slope when points are vertical—you only need to check the x-coordinate to see if it is on the line. And, in this case, the x-coordinate of X is the same as the other three points: 4. Either way you figure it, X is collinear with A, B, and C.

(b)
$$m(A, B) = \frac{5 - 7}{-\sqrt{7} - (-\sqrt{7})} = \frac{-2}{0}$$
so $m(A, B)$ is undefined.
$$m(A, X) = \frac{1 - 7}{-\sqrt{7} - (-\sqrt{7}} = \frac{-6}{0}$$
so $m(A, X)$ is also undefined.

The x-coordinate for X is the same as the x-coordinate for the other 3 points: $-\sqrt{7}$, and all the slopes are undefined, so X is collinear with A, B, and C.

(c)
$$m(A, B) = \frac{2 - 1}{0 - 0} = \frac{1}{0}$$
so $m(A, B)$ is undefined.
$$m(A, X) = \frac{0 - 1}{4 - 0} = \frac{-1}{4} = -\frac{1}{4}$$
so $m(A, X)$ is defined.

Also, the x-coordinates of points A, B, and C are all 0, but the x-coordinate of X is 4. Since the slopes between A, B, and C are all undefined, but $m(A, X)$ is defined, X cannot be collinear with A, B, and C.

(d)
$$m(A, B) = \frac{-2.9801 - \pi}{-\frac{14\pi}{23} - (-\frac{14\pi}{23})} = \frac{-2.9801 - \pi}{0}$$
so $m(A, B)$ is undefined.
$$m(A, X) = \frac{3\pi\sqrt{5} - \pi}{-\frac{14\pi}{23} - (-\frac{14\pi}{23})} = \frac{3\pi\sqrt{5} - \pi}{0}$$
so $m(A, X)$ is also undefined.

The x-coordinate for X is the same as the x-coordinate for the other 3 points: $-\frac{14\pi}{23}$, and all the slopes are undefined, so X is collinear with A, B, and C.

(e) In all four parts, the value of the x-coordinates of points A, B, and C is constant. In other words, A, B, and C lie on a vertical line.

16. (a)
$$m(A, B) = \frac{2 - 4}{1 - 2} = \frac{-2}{-1} = 2$$
$$m(A, X) = \frac{2 - 8}{1 - 4} = \frac{-6}{-3} = 2$$

Since $m(A, X) = m(A, B)$, X is collinear with A, B, and C.

(b)
$$m(A, B) = \frac{3 - 6}{1 - 2} = \frac{-3}{-1} = 3$$
$$m(A, X) = \frac{3 - (-12)}{1 - (-4)} = \frac{15}{5} = 3$$

Since $m(A, X) = m(A, B)$, X is collinear with A, B, and C.

(c)
$$m(A, B) = \frac{(-1) - (-2)}{8 - 16} = \frac{1}{-8} = -\frac{1}{8}$$
$$m(A, X) = \frac{(-1) - 8}{8 - 64} = \frac{-9}{-56} \neq -\frac{1}{8}$$

Since $m(A, X) \neq m(A, B)$, X is not collinear with A, B, and C.

(d)
$$m(A, C) = \frac{2 - 0}{-6 - 0} = \frac{2}{-6} = -\frac{1}{3}$$
$$m(C, X) = \frac{0 - \sqrt{2}}{0 - (-3\sqrt{2})} = \frac{-\sqrt{2}}{3\sqrt{2}} = -\frac{1}{3}$$

Since $m(A, X) = m(A, B)$, X is collinear with A, B, and C.

(e) In all four parts, B and C are "multiples" of A. In other words, in each part, if you multiply the x- and y-coordinates of point A by 2, you get point B. C is 3 times A. So, if X is a multiple of A, then it will be collinear with A, B and C.

4A MATHEMATICAL REFLECTIONS

1. If you pick any two points on a distance-time graph, the slope between the two points is the same as the average speed. This is because the vertical change on the distance-time graph represents *distance travelled* and the horizontal change is *time elapsed*. The slope between the two points is $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta\text{ distance}}{\Delta\text{ time}}$, which is the average speed.

2. (a) Collinear. All three points have the same y-coordinate, so they fall along the same horizontal line, the line $y = 2$.

(b) Not collinear. The ratio of the rise and run for points along the same line are proportional. The rise from D to E is $8 - 6 = 2$ and the run is $3 - 4 = -1$. The rise from E to F is $12 - 8 = 4$ and the run is $5 - 3 = -2$. Since

$$\frac{2}{-1} \neq \frac{4}{2}$$

the points D, E, and F must be on the same line.

(c) Not collinear. The ratio of the rise and run for points along the same line are proportional. The rise from G to H is $2 - 3 = -1$ and the run is $-1 - 0 = -1$. The rise from H to I is $7 - 2 = 5$ and the run is $2 - (-1) = 3$. Since

$$\frac{-1}{-1} \neq \frac{5}{3}$$

the points D, E, and F are not on the same line.

(d) Collinear. All three points haved the same x-coordinate, so they fall along the same vertical line, the line $x = -5$

3. (a) $m(A, B) = \frac{y-1}{2-(-4)} = \frac{y-1}{6}$ You want to find y such that

$$\begin{aligned} \frac{y-1}{6} &= 2 \\ 6 \cdot \frac{y-1}{6} &= 2 \cdot 6 \\ y - 1 &= 12 \\ y &= 13 \end{aligned}$$

(b) You want to find y such that

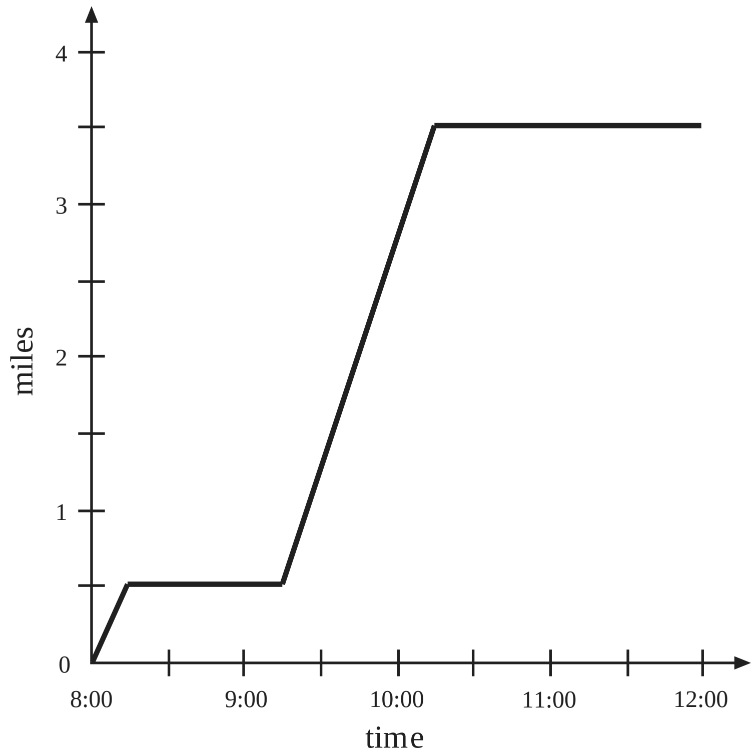
$$\begin{aligned} \frac{y-1}{6} &= -1 \\ 6 \cdot \frac{y-1}{6} &= -1 \cdot 6 \\ y - 1 &= -6 \\ y &= -5 \end{aligned}$$

(c) You want to find y such that

$$\begin{aligned} \frac{y-1}{6} &= 0 \\ 6 \cdot \frac{y-1}{6} &= 0 \cdot 6 \\ y - 1 &= 0 \\ y &= 1 \end{aligned}$$

(d) You want to find y such that $\frac{y-1}{6}$ does not exist. But, a fraction does not exist when the denominator is 0. You cannot make the denominator 0, so it is not possible to find y so that the slope does not exist.

4. (a)



(b) The distance is $\frac{1}{2}$ mile and the time is 15 minutes or $\frac{1}{4}$ hour. So, the average speed is $\frac{\Delta\text{ distance}}{\Delta\text{ time}} = \frac{\frac{1}{2}}{\frac{1}{4}} \frac{\text{mile}}{\text{hour}} = \frac{1}{2}\text{ mi} \div \frac{1}{4}\text{ hr} = \frac{1}{2} \cdot 4 \frac{\text{mi}}{\text{hr}} = 2$ miles per hour.

(c) The distance is 3 miles and the time is 1 hour. So, Chan's average speed is $\frac{\Delta\text{ distance}}{\Delta\text{ time}} = \frac{3\text{ miles}}{1\text{ hour}} = 3$ miles per hour.

(d) The distance is $\frac{1}{2} + 3 = 3\frac{1}{2} = \frac{7}{2}$ and the time is from 8:00 A.M. to 10:15 A.M. or $2\frac{1}{4} = \frac{9}{4}$. So, Chan's average speed is $\frac{\Delta\text{ distance}}{\Delta\text{ time}} = \frac{\frac{7}{2}}{\frac{9}{4}} \frac{\text{miles}}{\text{hour}} = \frac{7}{2} \div \frac{9}{4} \frac{\text{miles}}{\text{hour}} = \frac{7}{2} \cdot \frac{4}{9} \frac{\text{miles}}{\text{hour}} = \frac{14}{9} \frac{\text{miles}}{\text{hour}} = 1\frac{5}{9}$ miles per hour.

5. (a) Answers will vary. Choose a value for x and find the corresponding y value. If $x = 1$, then $y = 3(1) - 2 = 3 - 2 = 1$. If $x = -1$, then $y = 3(-1) - 2 = -3 - 2 = -5$. If $x = 3$, then $y = 3(3) - 2 = 9 - 2 = 7$. The three points are $A = (1, 1)$, $B = (-1, -5)$, and $C = (3, 7)$. To test for collinearity, find the slope.

$$\begin{aligned} m(A, B) &= \frac{-5-1}{-1-1} = \frac{-6}{-2} = 3 \\ m(A, C) &= \frac{7-1}{3-1} = \frac{6}{2} = 3 \end{aligned}$$

So, $m(A, B) = m(a, C)$. The points are collinear.

(b) Answers will vary. Three possible points are $D = (1, -3)$, $E = (-1, -7)$, and $F = (3, 1)$.

$$m(D,E) = \frac{-7-(-3)}{-1-1} = \frac{-7+3}{-2} = \frac{-4}{-2} = 2$$
$$m(E,F) = \frac{1-(-7)}{3-(-1)} = \frac{1+7}{3+1} = \frac{8}{4} = 2$$

So, $m(D,E) = m(E,F)$. The points are collinear.

(c) Answers will vary. Three possible points are $G = (1,4)$, $H = (-1,4)$, and $I = (3,28)$.

$$m(G,H) = \frac{4-4}{-1-1} = \frac{0}{-2} = 0$$
$$m(G,I) = \frac{28-4}{3-1} = \frac{24}{2} = 14$$

So, $m(G,H) \neq m(G,I)$. The points are not collinear.

(d) Answers will vary. Three possible points are $J = (1,0)$, $K = (-1,2)$, and $L = (3,-2)$.

$$m(J,K) = \frac{2-0}{-1-1} = \frac{2}{-2} = -1$$
$$m(K,L) = \frac{-2-2}{3-(-1)} = \frac{-4}{3+1} = \frac{-4}{4} = -1$$

So, $m(J,K) = m(K,L)$. The points are collinear.

6. Answers will vary. Some possible answers might be in finding the pitch of a roof, the slope of a ramp, or the designing of bridges.

7. Choose one of the points, and find the slope between that point and each of the other two points. If the slopes are equal, then the three points are collinear.

8. To see if the point $(3,-2)$ is on a line, find $m(A,B)$ and test if the slope from one of the points to $(3,-2)$ is equal to $m(A,B)$.

$$m(A,B) = \frac{3-(-5)}{4-2} = \frac{8}{2} = 4$$

The equation will be

$$\frac{-5-y}{2-x} = 4$$
$$\frac{-5-(-2)}{2-3} \stackrel{?}{=} 4$$
$$\frac{-3}{-1} \stackrel{?}{=} 4$$
$$3 \neq 4$$

So $(3,-2)$ is not on the line through A and B.

INVESTIGATION 4B LINEAR EQUATIONS AND GRAPHS

4.05 Getting Started

For You to Explore

1. The points that you select may vary, but no matter which two points you select, the slope between them will always be 3. The solution here demonstrates four sample points and the calculation of the slopes.

(a) The equation of the graph is $y = 3x$. To find a point, pick some values for x, plug them in, and get corresponding values for y. Two good numbers to start with for x are 0 and 1.

$$\begin{array}{ll} y = 3x & y = 3x \\ y = 3(0) & y = 3(1) \\ y = 0 & y = 3 \end{array}$$

So the points $(0,0)$ and $(1,3)$ are on the graph of $y = 3x$. The slope between the two points is

$$\frac{3-0}{1-0} = \frac{3}{1} = 3$$

so the slope between $(0,0)$ and $(1,3)$ is 3.

(b) To find two more points, start with two different values for x. This time, try -1 and 3.

$$\begin{array}{ll} y = 3x & y = 3x \\ y = 3(-1) & y = 3(3) \\ y = -3 & y = 9 \end{array}$$

So the points $(-1,-3)$ and $(3,9)$ are on the graph of $y = 3x$. The slope between these two points is

$$\frac{9-(-3)}{3-(-1)} = \frac{12}{4} = 3$$

so the slope between $(-1,-3)$ and $(3,9)$ is also 3.

(c) The slope between any two points on the line $y = 3x$ appears to be 3. Can you be sure? In the rest of this investigation, you will explore the relationship between slope and equations like $y = 3x$.

2. The points that you select may vary. The solution here demonstrates four sample points and the calculation of the slopes.

(a) The equation of the graph is $y = \frac{5}{x}$. To find a point, pick some values for x, plug them in, and get corresponding values for y. Two good numbers to start with for x are 1 and 5.

$$\begin{array}{ll} y = \frac{5}{x} & y = \frac{5}{x} \\ y = \frac{5}{(1)} & y = \frac{5}{(5)} \\ y = 5 & y = 1 \end{array}$$

So the points $(1,5)$ and $(5,1)$ are on the graph of $y = \frac{5}{x}$. The slope between the two points is

$$\frac{1-5}{5-1} = \frac{-4}{4} = -1$$

so the slope between $(1,5)$ and $(5,1)$ is -1.

(b) To find two more points, start with two different values for x. This time, try 2 and 3.

$$\begin{array}{ll} y = \frac{5}{x} & y = \frac{5}{x} \\ y = \frac{5}{(2)} & y = \frac{5}{(3)} \\ y = \frac{5}{2} & y = \frac{5}{3} \end{array}$$

So the points $\left(2, \frac{5}{2}\right)$ and $\left(3, \frac{5}{3}\right)$ are on the graph of $y = \frac{5}{x}$. The slope between these two points is

$$\frac{\frac{5}{3} - \frac{5}{2}}{3 - 2} = \frac{-\frac{5}{6}}{4} = -\frac{5}{24}$$

so the slope between $\left(2, \frac{5}{2}\right)$ and $\left(3, \frac{5}{3}\right)$ is $-\frac{5}{24}$.

(c) Unlike Exercise 1, the two slopes were not equal.

3. The points that you select may vary. The solution here demonstrates four sample points and the calculation of the slopes.

(a) The equation of the graph is $y = 2x^2$. To find a point, pick some values for x, plug them in, and get corresponding values for y. Two good numbers to start with for x are 1 and 2.

$$\begin{array}{ll} y = 2x^2 & y = 2x^2 \\ y = 2((1)^2) & y = 2((2)^2) \\ y = 2 & y = 8 \end{array}$$

So the points $(1, 2)$ and $(2, 8)$ are on the graph of $y = 2x^2$. The slope between the two points is

$$\frac{8 - 2}{2 - 1} = \frac{6}{1} = 6$$

so the slope between $(0, 0)$ and $(1, 2)$ is 6.

(b) To find two more points, start with two different values for x. This time, try 3 and 4.

$$\begin{array}{ll} y = 2x^2 & y = 2x^2 \\ y = 2((3)^2) & y = 2((4)^2) \\ y = 18 & y = 32 \end{array}$$

So the points $(3, 18)$ and $(4, 32)$ are on the graph of $y = 2x^2$. The slope between these two points is

$$\frac{32 - 18}{4 - 3} = \frac{14}{1} = 14$$

so the slope between $(2, 8)$ and $(3, 18)$ is 14.

(c) As in Exercise 2, the two slopes were not equal.

4. The points that you select may vary, but no matter which two points you select from the first quadrant, the slope between them will always be 4. Parts (a), (b), and (c) demonstrate four sample points from the first quadrant and the calculation of the slopes.

(a) The equation of the graph is $y = |4x|$. To find a point, pick some values for x, plug them in, and get corresponding values for y. Two good numbers to start with for x are 1 and 2.

$$\begin{array}{ll} y = |4x| & y = |4x| \\ y = |4(1)| & y = |4(2)| \\ y = |4| & y = |8| \\ y = 4 & y = 8 \end{array}$$

So the points $(1, 4)$ and $(2, 8)$ are on the graph of $y = |4x|$. The slope between the two points is

$$\frac{8 - 4}{2 - 1} = \frac{4}{1} = 4$$

so the slope between $(1, 4)$ and $(2, 8)$ is 4.

(b) To find two more points, start with two different values for x. This time, try 3 and 4.

$$\begin{array}{ll} y = |4x| & y = |4x| \\ y = |4(3)| & y = |4(4)| \\ y = |12| & y = |16| \\ y = 12 & y = 16 \end{array}$$

So the points $(3, 12)$ and $(4, 16)$ are on the graph of $y = |4x|$. The slope between these two points is

$$\frac{16 - 12}{4 - 3} = \frac{4}{1} = 4$$

so the slope between $(3, 12)$ and $(4, 16)$ is also 4.

(c) The slope between any two points on the line $y = |4x|$ appears to be 4. Can you be sure?

(d) To find two points in the second quadrant, start with negative values for x, like -1 and -2.

$$\begin{array}{ll} y = |4x| & y = |4x| \\ y = |4(-1)| & y = |4(-2)| \\ y = |-4| & y = |-8| \\ y = 4 & y = 8 \end{array}$$

So the points $(-1, 4)$ and $(-2, 8)$ are on the graph of $y = |4x|$. The slope between the two points is

$$\frac{8 - 4}{(-2) - (-1)} = \frac{4}{-1} = -4$$

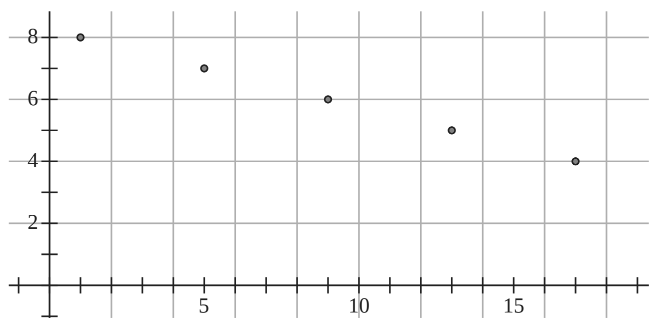
so the slope between $(-1, 4)$ and $(-2, 8)$ is -4. So, for the equation $y = |4x|$, the slope between two points in the first quadrant seems to always be 4, but the slope between points in the second quadrant seems to be -4.

5. You can use the slope to find points that are collinear to these two. Remember that slope is "rise over run," which means that you add the numerator to the y-coordinate, and add the denominator to the x-coordinate. If the slope is negative, you can put the negative sign with either the numerator or denominator, *but not both*.

(a) The slope between the two given points is

$$\frac{8 - 7}{1 - 5} = \frac{1}{-4} = -\frac{1}{4}$$

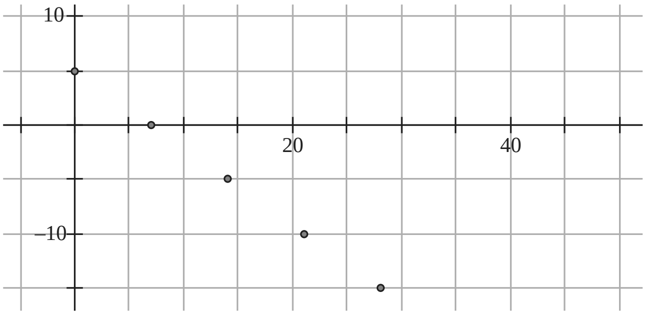
Find additional points by adding -1 to the y-coordinate, and 4 to the x-coordinate:
$(5 + 4, 7 - 1) = (9, 6)$
$(9 + 4, 6 - 1) = (13, 5)$
$(13 + 4, 5 - 1) = (17, 4)$



(b) The slope is

$$\frac{5-0}{0-7} = \frac{5}{-7} = -\frac{5}{7}$$

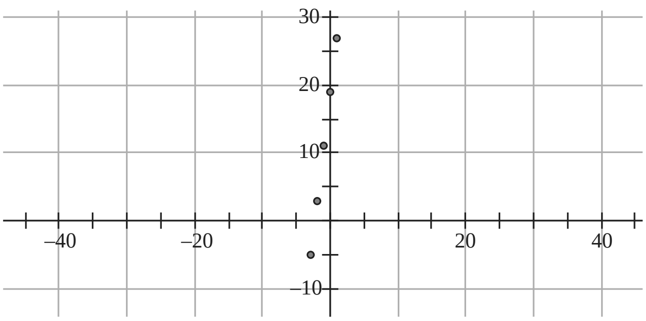
Additional points are
$(7+7, 0-5) = (14, -5)$
$(14+7, -5-5) = (21, -10)$
$(21+7, -10-5) = (28, -15)$



(c) The slope is

$$\frac{(-5)-3}{(-3)-(-2)} = \frac{-8}{-1} = 8$$

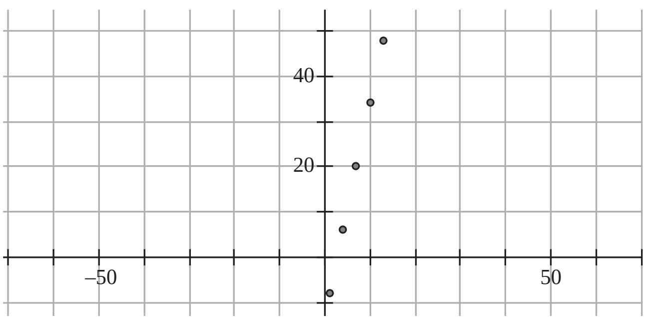
Additional points are
$(-2+1, 3+8) = (-1, 11)$
$(-1+1, 11+8) = (0, 19)$
$(0+1, 19+8) = (1, 27)$



(d) The slope is

$$\frac{6-(-8)}{4-1} = \frac{14}{3}$$

Additional points are
$(4+3, 6+14) = (7, 20)$
$(7+3, 20+14) = (10, 34)$
$(10+3, 34+14) = (13, 48)$



6. For each part, you should already have figured out the slope between the two points in Exercise 5. Your point tester should check that the slope between $P = (x, y)$ and one of the two points is equal to the slope you already calculated.

(a) The slope is $-\frac{1}{4}$. To test that the slope from the point $(5, 7)$ to (x, y) is $-\frac{1}{4}$, use the equation

$$\frac{y-7}{x-5} = -\frac{1}{4}$$

(b) The slope is $-\frac{5}{7}$. The point-tester then is

$$\frac{y}{x-7} = -\frac{5}{7}$$

(c) The slope is 8. The point-tester then is

$$\frac{y-3}{x-(-2)} = 8$$

which simplifies to

$$\frac{y-3}{x+2} = 8$$

(d) The slope is $\frac{14}{3}$. The point-tester then is

$$\frac{y+8}{x-1} = \frac{14}{3}$$

On Your Own

7. (a) At 8:10 A.M., John has ridden 10 minutes. If John rides 15 ft/s, he rides $15 \text{ ft/s} \cdot 60 \text{ s/min} = 900 \text{ ft/min}$. So after 10 minutes, John has ridden $900 \text{ ft/min} \cdot 10 \text{ min} = 9000$ feet, or about 1.7 miles.

(b) After 20 minutes, John has ridden $900 \text{ ft/min} \cdot 20 \text{ min} = 18{,}000$ feet, or about 3.4 miles.

(c) After 45 minutes, John has ridden $900 \text{ ft/min} \cdot 45 \text{ min} = 40{,}500$ feet, or about 7.7 miles.

(d) After t minutes, John has ridden $900 \text{ ft/min} \cdot t \text{ min} = 900t$ feet.

8. (a) If Pete leaves 10 minutes after John, he leaves at 8:10 A.M. Pete has not ridden any distance yet, so he hasn't caught John.

(b) At 8:20 A.M., Pete has ridden for 10 minutes. If Pete rides 20 ft/sec, he rides $20 \text{ ft/s} \cdot 60 \text{ s/min} = 1200$ ft/min. So after 10 minutes, Pete has ridden $1200 \text{ ft/min} \cdot 10 \text{ min} = 12{,}000$ feet, or about $2\frac{1}{4}$ miles. Pete still has not caught John, since John has ridden 3.4 miles by 8:20 A.M.

(c) At 8:45 A.M., Pete has ridden for 35 minutes and gone $1200 \text{ ft/min} \cdot 35 \text{ min} = 42{,}000$ feet, or just about 8 miles. Pete has ridden further than John, so he must have passed him before 8:45.

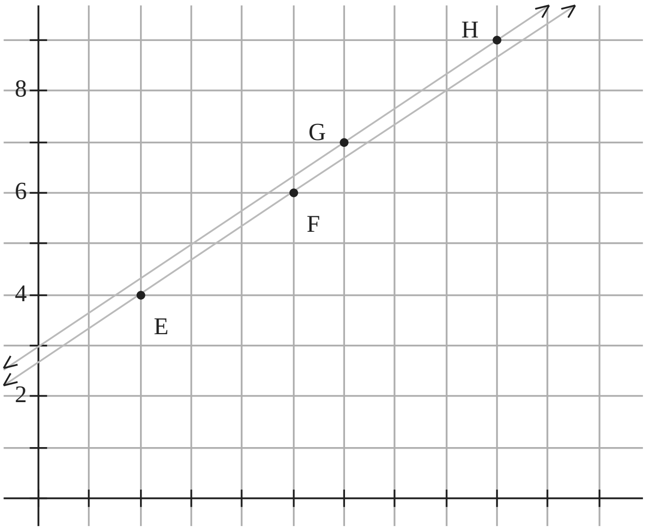
(d) At t minutes after 8:00 A.M., Pete has only ridden for $t-10$ minutes, so he has ridden $1200 \cdot (t-10)$ feet.

(e) The exact time Pete caught John would be the time when the distance they have ridden is equal. So, say that time is t. John has gone $900t$ feet, and Pete has gone $1200(t-10)$ feet. Set them equal, and solve for t.

$$
\begin{aligned}
900t &= 1200(t-10) \\
900t &= 1200t - 12000 \\
900t - 1200t &= -12000 \\
-300t &= -12000 \\
t &= \frac{-12000}{-300} \\
t &= 40
\end{aligned}
$$

So Pete passes John 40 minutes after John has started, which would be 8:40 A.M.

9. (a)



(b) The assumption on page 343 says to compare three points, not four. Most importantly, the two calculations of slope must have one point in common. After calculating $m(E, F)$, if he calculated, say, $m(F, G)$ and got the same slope, then he could say E, F, and G were collinear. He would then have to test H with one of the three points to say all four were collinear.

(c) The line that contains E and F and the line that contains G and H are parallel.

10. The points that you found may vary from the points given here, but the method for finding them, and the way they align on the sketch, should be roughly the same.

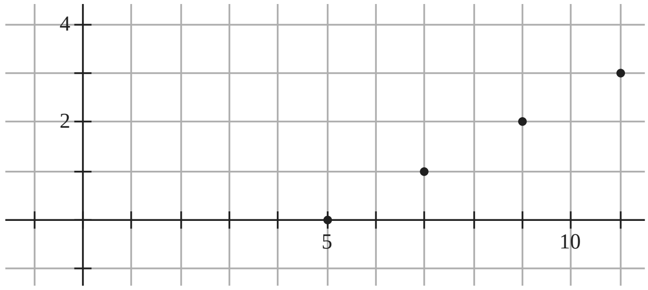
(a) The easiest way to find a point such that the slope from that point to $(5, 0)$ is $\frac{1}{2}$ is to use the concept of "rise over run." Start with $(5, 0)$, then rise by 1, which means to add 1 to the y-coordinate, and run by 2, meaning add 2 to the x-coordinate, to get $(5 + 2, 0 + 1) = (7, 1)$. You can double-check the method by testing the slope between $(5, 0)$ and $(7, 1)$

$$\frac{1-0}{7-5} = \frac{1}{2}$$

Find 2 more points by repeating the process:

- $(7 + 2, 1 + 1) = (9, 2)$
- $(9 + 2, 2 + 1) = (11, 3)$

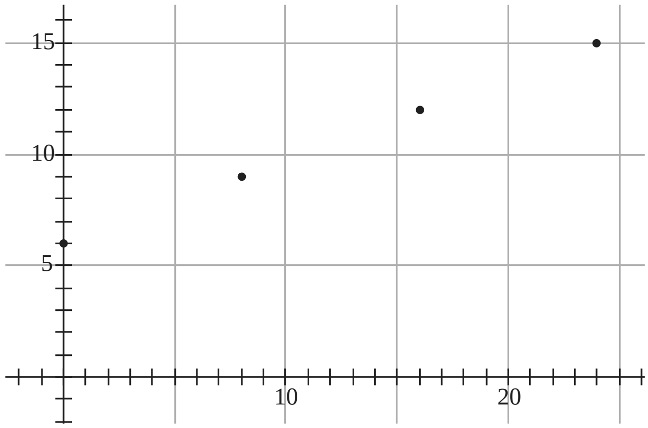
These four points have the same slope between each other, so they are collinear.



(b) Follow the same procedure as part (a).

- $(0 + 8, 6 + 3) = (8, 9)$
- $(8 + 8, 9 + 3) = (16, 12)$
- $(16 + 8, 12 + 3) = (24, 15)$

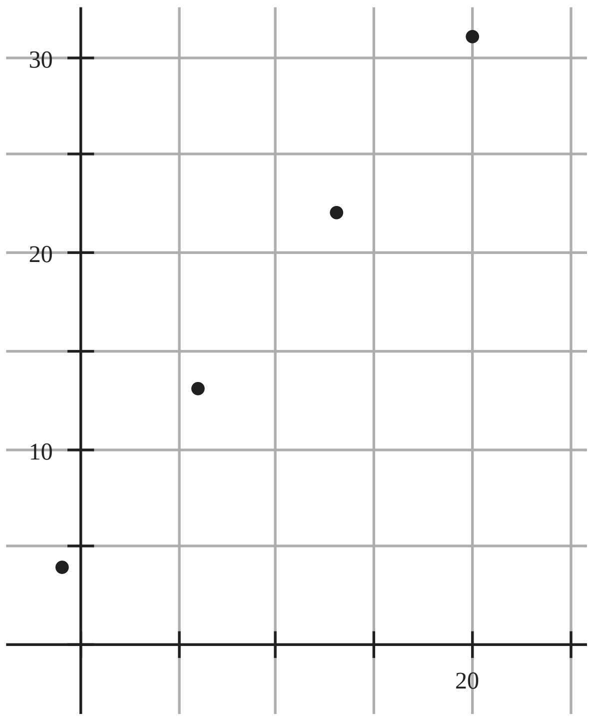
These four points have the same slope between each other, so they are collinear.



(c) Follow the same procedure as part (a).

- $(-1 + 7, 4 + 9) = (6, 13)$
- $(6 + 7, 13 + 9) = (13, 22)$
- $(13 + 7, 22 + 9) = (20, 31)$

These four points have the same slope between each other, so they are collinear.



(d) The slope for this part is negative. You probably think of "rise" as "go up," and you've seen "run" to mean "go to the right." With a negative slope, one of the two has to go in the opposite direction. In this case, you can choose for the rise to go *down*.

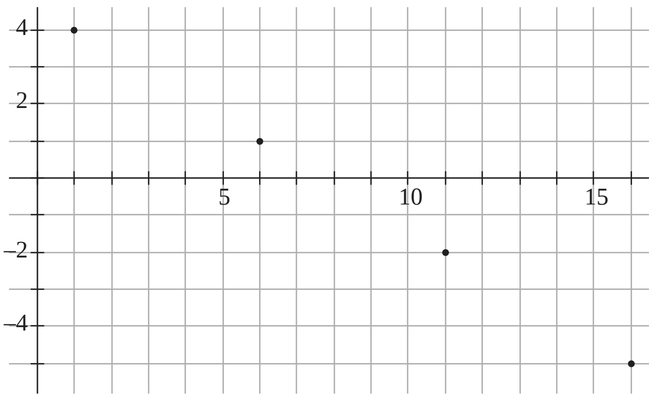
So, to find one point, start with $(1, 4)$, then rise by -3, which means, as before, to add -3 to the y-coordinate, and run by 5 to get $(1+5, 4+(-3)) = (6, 1)$. You can double-check the method by testing the slope between $(1, 4)$ and $(6, 1)$

$$\frac{1-4}{6-1} = \frac{-3}{5} = -\frac{3}{5}$$

Again, find 2 more points by repeating the process:

- $(6+5, 1+(-3)) = (11, -2)$
- $(11+5, -2+(-3)) = (16, -5)$

These four points have the same slope between each other, so they are collinear.



11. (a) A point-tester will test whether the slope between point (x, y) and point $(5, 0)$ is equal to $\frac{1}{2}$. So, just substitute the two points into the slope formula, and set it equal to the slope. (Remember, $y = 0$ is equal to just y.)

$$\frac{y}{x-5} = \frac{1}{2}$$

(b) $\frac{y-6}{x} = \frac{3}{8}$

(c) Remember, $x - (-1)$ is equivalent to $x + 1$. $\frac{y-4}{x+1} = \frac{9}{7}$

(d) $\frac{y-4}{x-1} = -\frac{3}{5}$

12. (a)

$$\begin{aligned} y+3 &= 2(x-5) \\ y+3 &= 2x-10 \\ y+3-3 &= 2x-10-3 \\ y &= 3x-13 \end{aligned}$$

(b)

$$\begin{aligned} y-2 &= -\frac{1}{3}(x+3) \\ y-2 &= -\frac{1}{3}x-1 \\ y-2+2 &= -\frac{1}{3}x-1+2 \\ y &= -\frac{1}{3}x+1 \end{aligned}$$

(c)

$$\begin{aligned} 4x-5y &= 7 \\ 4x-5y-4x &= 7-4x \\ -5y &= -4x+7 \\ \left(-\frac{1}{5}\right)(-5y) &= \left(-\frac{1}{5}\right)(-4x+7) \\ y &= \frac{4}{5}x-\frac{7}{5} \end{aligned}$$

(d)

$$\begin{aligned} \frac{y-8}{x} &= -3 \\ \frac{y-8}{x}\cdot x &= -3\cdot x \\ y-8 &= -3x \\ y-8+8 &= -3x+8 \\ y &= -3x+8 \end{aligned}$$

(e)

$$\begin{aligned} y+6 &= 2x+1 \\ y+6-6 &= 2x+1-6 \\ y &= 2x-5 \end{aligned}$$

(f)

$$\begin{aligned} \frac{y-2}{x-1} &= 5 \\ \frac{y-2}{x-1}\cdot(x-1) &= 5\cdot(x-1) \\ y-2 &= 5x-5 \\ y-2+2 &= 5x-5+2 \\ y &= 5x-3 \end{aligned}$$

(g)

$$\begin{aligned} \frac{y+7}{x-3} &= -\frac{11}{3} \\ \frac{y+7}{x-3}\cdot(x-3) &= -\frac{11}{3}\cdot(x-3) \\ y+7 &= -\frac{11}{3}x+11 \\ y+7-7 &= -\frac{11}{3}x+11+7 \\ y &= -\frac{11}{3}x+18 \end{aligned}$$

(h)

$$\begin{aligned} xy &= 9 \\ xy\cdot\frac{1}{x} &= 9\cdot\frac{1}{x} \\ y &= \frac{9}{x} \end{aligned}$$

Maintain Your Skills

13. (a) The slope between $(1, 2)$ and $(3, 3)$ is

$$\frac{3-2}{3-1} = \frac{1}{2}$$

(b) $(2, 3)$ and $(4, 4)$:

$$\frac{4-3}{4-2} = \frac{1}{2}$$

(c) $(3, 4)$ and $(5, 5)$:

$$\frac{5-4}{5-3} = \frac{1}{2}$$

(d) $(4, 5)$ and $(6, 6)$:

$$\frac{6-5}{6-4} = \frac{1}{2}$$

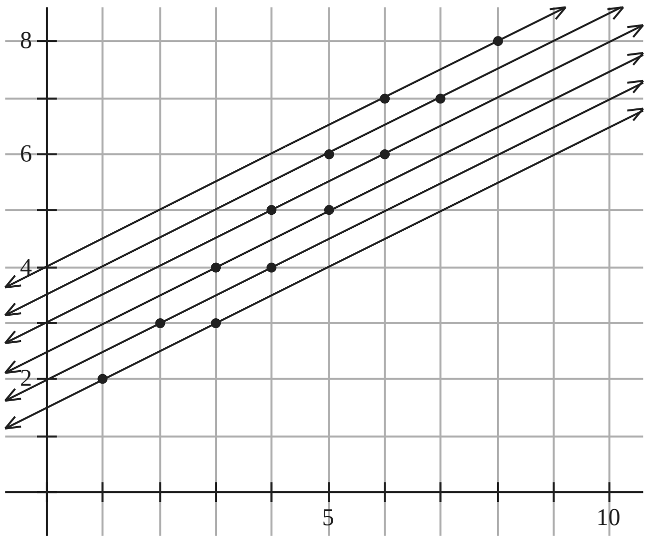
(e) $(5, 6)$ and $(7, 7)$:

$$\frac{7-6}{7-5} = \frac{1}{2}$$

(f) $(6, 7)$ and $(8, 8)$:

$$\frac{8-7}{8-6} = \frac{1}{2}$$

(g) The slope between each pair of points is $\frac{1}{2}$. The points are not all collinear, but the lines that contain each pair of points are parallel.



(h) The slope between $(1+a, 2+a)$ and $(3+a, 3+a)$ is

$$\begin{aligned}\frac{(3+a)-(2+a)}{(3+a)-(1+a)} &= \frac{3+a-2-a}{3+a-1-a} \\ &= \frac{3-2+a-a}{3-1+a-a} \\ &= \frac{3-2}{3-1} \\ &= \frac{1}{2}\end{aligned}$$

Each of the pairs of points is in the form $(1+a, 2+a)$ and $(3+a, 3+a)$, so no matter what a is, the slope will be $\frac{1}{2}$.

(i) From part (a), you know that $m(A, B) = \frac{1}{2}$.

$$\begin{aligned}m(A', B') &= \frac{(3+b)-(2+b)}{(3+a)-(1+a)} \\ &= \frac{3+b-2-b}{3+a-1-a} \\ &= \frac{3-2+b-b}{3-1+a-a} \\ &= \frac{3-2}{3-1} \\ &= \frac{1}{2}\end{aligned}$$

So no matter what the values are for a and b, if you translate points using the transformation rule $(x, y) \mapsto (x+a, y+b)$, the slope between points will stay the same.

14. (a) The slope between $(1, 2)$ and $(3, 3)$ is

$$\frac{3-2}{3-1} = \frac{1}{2}$$

(b) $(2, 4)$ and $(6, 6)$:

$$\frac{6-4}{6-2} = \frac{2}{4} = \frac{1}{2}$$

(c) $(3, 6)$ and $(9, 9)$:

$$\frac{9-6}{9-3} = \frac{3}{6} = \frac{1}{2}$$

(d) $(4, 8)$ and $(12, 12)$:

$$\frac{12-8}{12-4} = \frac{4}{8} = \frac{1}{2}$$

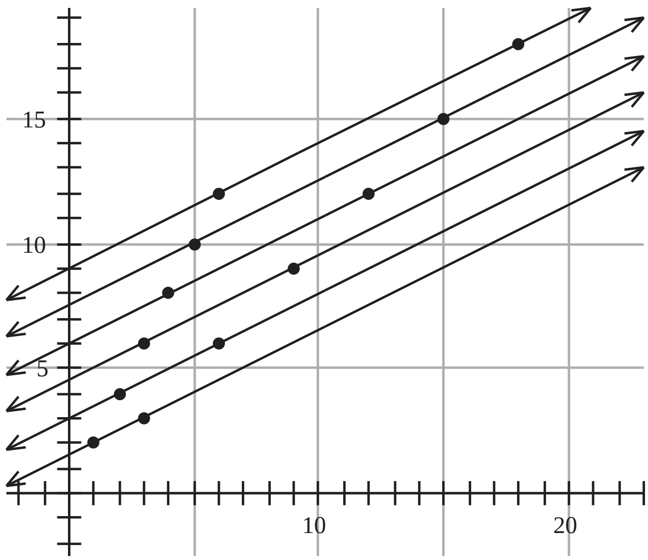
(e) $(5, 10)$ and $(15, 15)$:

$$\frac{15-10}{15-5} = \frac{5}{10} = \frac{1}{2}$$

(f) $(6, 12)$ and $(18, 18)$:

$$\frac{18-12}{18-6} = \frac{6}{12} = \frac{1}{2}$$

(g) The slope between each pair of points is $\frac{1}{2}$. While the points are not all collinear, the lines that contain each pair of points are parallel to each other.



(h) The slope between $(a, 2a)$ and $(3a, 3a)$ is

$$\frac{3a-2a}{3a-a} = \frac{a}{2a} = \frac{1}{2}$$

Each of the pairs of points is in the form $(a, 2a)$ and $(3a, 3a)$, so no matter what a is, the slope will be $\frac{1}{2}$.

(i) From part (a), you know that $m(A, B) = \frac{1}{2}$. So

$$m(A', B') = \frac{3b-2b}{3a-a} = \frac{b}{2a}$$

In this case, the transformation preserves the slope only if $a = b$. In other words, if you apply the transformation $(x, y) \mapsto (ax, ay)$ to two points, the slope between the slopes will stay the same. But, if you apply the rule $(x, y) \mapsto (ax, by)$ where $a \neq b$, the slope will change.

4.06 Equations of Lines

Check Your Understanding

1. Using $S = (6, 4)$ as the base point, the point-tester equation will be

$$\frac{y-2}{x-6} = -\frac{1}{4}$$

You can then multiply both sides by $(x - 6)$ to get

$$y - 2 = -\frac{1}{4}(x-6)$$

This equation is not exactly the same as the one Sasha and Tony came up with. You can show they are *equivalent* by solving both equations for y, and simplifying the right side as much as possible. That way, both equations should end up being the same if they are equivalent.

$$\begin{aligned} y - 2 &= -\frac{1}{4}(x-6) & \qquad y - 4 &= -\frac{1}{4}(x+2) \\ y &= -\frac{1}{4}(x-6) + 2 & y &= -\frac{1}{4}(x+2) + 4 \\ y &= -\frac{1}{4}x + \frac{3}{2} + 2 & y &= -\frac{1}{4}x - \frac{1}{2} + 4 \\ y &= -\frac{1}{4}x + \frac{7}{2} & y &= -\frac{1}{4}x + \frac{7}{2} \end{aligned}$$

Since the two equations could be transformed using the basic rules and moves to the same equation, they must be equivalent.

2. Check students' work.

3. (a) First calculate $m(A, B)$:

$$m(A, B) = \frac{7-1}{6-12} = \frac{6}{-6} = -1$$

Now, using $A = (6, 7)$ as a base point, use a point-tester equation. So, $P = (x, y)$ is on the line if

$$\begin{aligned} m(P, A) &= -1 \\ \frac{y-7}{x-6} &= -1 \\ y - 7 &= -1(x-6) \end{aligned}$$

(b) First calculate the slope between the two points:

$$\frac{4-(-4)}{5-(-3)} = \frac{8}{8} = 1$$

So, using $(5, 4)$ as a base point, (x, y) is on the line if

$$\begin{aligned} \frac{y-4}{x-5} &= 1 \\ y - 4 &= x - 5 \end{aligned}$$

(c) The origin is $(0, 0)$. Calculate the slope between the two points:

$$\frac{3-0}{9-0} = \frac{1}{3}$$

Using the origin as a base point, (x, y) is on the line if

$$\begin{aligned} \frac{y-0}{x-0} &= \frac{1}{3} \\ y &= \frac{1}{3}x \end{aligned}$$

(d) The line will have the same slope as the line in part (c), $\frac{1}{3}$. Using $(0, 10)$ as a base point, a point will be on the line if

$$\begin{aligned} \frac{y-10}{x-0} &= \frac{1}{3} \\ y - 10 &= \frac{1}{3}x \end{aligned}$$

(e) The line will have the same slope as the line in part (a), -1. Using the origin $(0, 0)$ as a base point, a point will be on the line if

$$\begin{aligned} \frac{y-0}{x-0} &= -1 \\ y &= -x \end{aligned}$$

4. (a) One way to solve the problem is to find a second point on the line. If you draw a picture, you can see that $(0, 7)$ is also on the line. To write a point tester for the graph, start out by calculating the slope between these two points:

$$\frac{7-7}{6-0} = 0$$

So, the slope of the line is 0. Now, using $(6, 7)$ as a base point, use a point-tester equation:

$$\begin{aligned} \frac{y-7}{x-6} &= 0 \\ y - 7 &= 0 \end{aligned}$$

Another way to solve the problem would be to inspect the graph and try to find a way to characterize all of the points of the graph, which is that the y-coordinate is always 7, so your equation would be $y = 7$, which is equivalent to the equation above.

(b) If you follow the same process to calculate slope, you hit a dead end:

$$\frac{-4-4}{5-5} = \frac{8}{0}$$

which is undefined. So, sketch a graph and see that the graph is a vertical line. Both the given points, and all the other points on the line, have 5 as their x-coordinate, so the equation of the line is

$$x = 5$$

(c) First, carefully draw the line that fits the description. Start looking at the points on the line. Some of the points are $(1, 1)$, $(2, 2)$, $(5, 5)$, $(-2, -2)$, and $(-4, -4)$. Notice that for every point, the x- and y-coordinates are the same. In other words,

$$x = y$$

You can also pick two points, find the slope between them, and build a point-tester. Choose any two points, say, the origin $(0, 0)$ and $(1, 1)$.

$$\frac{1-0}{1-0} = \frac{1}{1} = 1$$

Using $(0, 0)$ as the base point, the point-tester equation is

$$\begin{aligned} \frac{y-0}{x-0} &= 1 \\ y &= x \end{aligned}$$

(d) Parallel lines have the same slope. In part (a), you calculated the slope to be 0. So, the line goes through the origin and has slope 0. Using $(0, 0)$ as the base point, the point-tester equation is

$$\begin{aligned} \frac{y-0}{x-0} &= 0 \\ y &= 0 \end{aligned}$$

The result is the x-axis, the horizontal line separating positive and negative values of y.

5. The quickest way to tell if the graph of an equation is a line is to look at the variables:

- there will be two (in this case, x and y)
- there are no powers of the variables (for example, no x^2 or y^{12}, just x and y)
- the variables are not multiplied together (so you see no xy)

Also, you want to make sure that if there are variables in the denominator of a fraction (like $\frac{y}{x-2}$), you multiply both sides by the denominator first to clear it out before checking. If these checks are too hard to see, use the basic rules and moves to change the equation to one that is in the form $ax + by = c$ or $y = ax + b$.

(a) The equation seems to fit the rules outlined above. To make sure, solve for y to see if the equation is then in the form $y = ax + b$.

$$\begin{aligned} y - 3 &= \frac{3}{4}(x-8) \\ y - 3 &= \frac{3}{4}x - 6 \\ y &= \frac{3}{4}x - 3 \end{aligned}$$

So, in this case, the graph of the equation is indeed a line.
When the line crosses the x-axis, the y-coordinate will equal 0. So, plug $y = 0$ into the equation of the line:

$$\begin{aligned} y &= \frac{3}{4}x - 3 \\ (0) &= \frac{3}{4}x - 3 \\ 3 &= \frac{3}{4}x \\ 4 &= x \end{aligned}$$

So, $(4, 0)$ is where the line crosses the x-axis (the x-intercept).
When the line crosses the y-axis, the x-coordinate will equal 0. So, plug $x = 0$ into the equation of the line:

$$\begin{aligned} y &= \frac{3}{4}x - 3 \\ y &= \frac{3}{4}(0) - 3 \\ y &= -3 \end{aligned}$$

So, $(0, -3)$ is where the line crosses the y-axis (the y-intercept).
You can use these two points to figure the slope:

$$\frac{0-(-3)}{4-0} = \frac{3}{4}$$

So, in summary,

- The graph of the equation is a line.
- slope = $\frac{3}{4}$
- x-intercept = $(4, 0)$
- y-intercept = $(0, -3)$

(b) Right away, notice that one of the rules outlined above is broken: one term has xy, so x and y are multiplied together. To make sure that the graph of this equation is not a line, find three points that are on the graph but are not collinear. The points $A = (3, 4)$, $B = (2, 6)$, and $C = (6, 2)$ all satisfy the equation.

$$\begin{aligned} m(A, B) &= \frac{4-6}{3-2} & m(A, C) &= \frac{4-2}{3-6} \\ &= \frac{-2}{1} & &= \frac{2}{-3} \\ &= -2 & &= -\frac{2}{3} \end{aligned}$$

Since $m(A, B) \neq m(A, C)$ these points are not collinear. Since all three of these points were on the graph of the equation, the graph of the equation is not a line.

(c) You may recognize this equation as the one you got when you simplified the one in part (a). In any case, the equation is of the form $y = ax + b$, and it passes all the tests, so it is a line. Also, since the equation is equivalent to the one in part (a), the slope and intercepts will also be the same (since they will have the same graph!).

(d) This equation also passes all the tests, and is already solved for y and in the form $y = ax + b$, so it is a line.

The line crosses the x-axis when the y-coordinate is 0:

$$\begin{aligned}(0) &= 5x - 7\\ 7 &= 5x\\ \frac{7}{5} &= x\end{aligned}$$

So, $\left(\frac{7}{5}, 0\right)$ is the where the line crosses the x-axis (the x-intercept).
The line crosses the y-axis when the x-coordinate is 0.

$$\begin{aligned}y &= 5(0) - 7\\ y &= -7\end{aligned}$$

So, $(0, -7)$ is where the line crosses the y-axis (the y-intercept). The slope between $\left(\frac{7}{5}, 0\right)$ and $(0, -7)$ is

$$\frac{0-(-7)}{\frac{7}{5}-0} = \frac{7}{\frac{7}{5}} = \frac{7}{1}\cdot\frac{5}{7} = 5$$

So, to summarize,

- The graph of the equation is a line.
- slope $= 5$
- x-intercept $= \left(\frac{7}{5}, 0\right)$
- y-intercept $= (0, -7)$

(e) The equation is in the form $ax + by = c$, so its graph must be a line.
x-intercept:
$$\begin{aligned}3x - 4(0) &= 12\\ 3x &= 12\\ x &= 4\end{aligned}$$

So the x-intercept is $(4, 0)$.
y-intercept:
$$\begin{aligned}3(0) - 4y &= 12\\ -4y &= 12\\ y &= -3\end{aligned}$$

So the y-intercept is $(0, -3)$.
Notice that the x- and y-intercepts are the same as parts (a) and (c), so this equation must be equivalent to both of those two (if you solve this equation for y, you will get equation (c)). So the answers are the same as parts (a) and (c).

(f) Again, the equation passes all the tests, and is in the form $y = ax + b$, so it must be a line.
x-intercept:
$$\begin{aligned}(0) &= -\frac{5}{3}x - 9\\ 9 &= -\frac{5}{3}x\\ 9\left(-\frac{3}{5}\right) &= x\\ -\frac{27}{5} &= x\end{aligned}$$

So $\left(-\frac{27}{5}, 0\right)$ is the x-intercept.
y-intercept:
$$\begin{aligned}y &= -\frac{5}{3}(0) - 9\\ y &= -9\end{aligned}$$

So $(0, -9)$ is the y-intercept.

The slope between $\left(-\frac{27}{5}, 0\right)$ and $(0, -9)$ is

$$\frac{0-(-9)}{-\frac{27}{5}-0} = \frac{9}{-\frac{27}{5}} = \frac{9}{1}\cdot\frac{-5}{27} = -\frac{5}{3}$$

To summarize,

- The graph of the equation is a line.
- slope = $-\frac{5}{3}$.
- x-intercept = $\left(-\frac{27}{5}, 0\right)$
- y-intercept = $(0, -9)$

(g) The graph of the equation is not a line (it is a circle, in fact). It has the terms x^2 and y^2, which break the “no powers” test.

To make sure that the graph of the equation cannot be a line, find three points on the graph and show that they are not collinear. Three points that are simpler to find are $A = (1, 0)$, $B = (-1, 0)$, and $C = (0, 1)$.

$$\begin{aligned}m(A, B) &= \frac{0-0}{1-(-1)} & m(A, C) &= \frac{0-1}{1-0}\\ &= \frac{0}{2} & &= \frac{-1}{1}\\ &= 0 & &= -1\end{aligned}$$

Since $m(A, B) \neq m(A, C)$, A, B, and C are not collinear, and the graph of this equation is not a line.

(h) The equation is in the form $ax + by = c$, so its graph is a line.
x-intercept:
$$\begin{aligned}3x - 4(0) &= 9\\ 3x &= 9\\ x &= 3\end{aligned}$$

So $(3, 0)$ is the x-intercept.
y-intercept:
$$\begin{aligned}3(0) - 4y &= 9\\ -4y &= 9\\ y &= -\frac{9}{4}\end{aligned}$$

So $\left(0, -\frac{9}{4}\right)$ is the y-intercept.
The slope between these two points is

$$\frac{0-\left(-\frac{9}{4}\right)}{3-0} = \frac{\frac{9}{4}}{3} = \frac{9}{4}\cdot\frac{1}{3} = \frac{3}{4}$$

To summarize,

- The graph of the equation is a line.
- slope = $\frac{3}{4}$
- x-intercept = $(3, 0)$
- y-intercept = $\left(0, -\frac{9}{4}\right)$

(i) The equation passes all the tests, and it is in the form $y = ax + b$, so its graph will be a line.
x-intercept:
$$\begin{aligned}(0) &= -\frac{5}{3}x + 8\\ \frac{5}{3}x &= 8\\ x &= \frac{3}{5}(8)\\ x &= \frac{24}{5}\end{aligned}$$

So $\left(\frac{24}{5}, 0\right)$ is the x-intercept.

y-intercept:

$$y = -\frac{5}{3}(0) + 8$$
$$y = 8$$

So $(0, 8)$ is the y-intercept.
The slope between $\left(\frac{24}{5}, 0\right)$ and $(0, 8)$ is

$$\frac{0-8}{\frac{24}{5}-0} = \frac{-8}{\frac{24}{5}} = \frac{-8}{1} \cdot \frac{5}{24} = -\frac{5}{3}$$

To summarize,

- The graph of the equation is a line.
- slope = $-\frac{5}{3}$
- x-intercept =$\left(\frac{24}{5}, 0\right)$
- y-intercept = $(0, 8)$

A few things you might notice from this exercise:

- The slopes of the lines in part (f) and part (i) are the same.
- In parts (c), (f), and (i), the original equation was in the form $y = ax + b$. It turns out that the slope was equal to a, the coefficient of x.
- In the same three parts, the y-coordinate of the y-intercept showed up as the b from the form $y = ax + b$.

6. To show that the slope between any two points will always be 3, you first need to find a way to express two arbitrary points. The hint in the sidenote should help. If the x-coordinate for one point is, say, a, then the y-coordinate would be

$$\begin{aligned} 3x - y &= 7 \\ 3(a) - y &= 7 \\ -y &= -3a + 7 \\ y &= 3a - 7 \end{aligned}$$

So the point $(a, 3a-7)$ is on the graph of $3x - y = 7$. Call that point A. A second point, B, could be written as $(b, 3b-7)$. Now, find the slope between A and B:

$$\begin{aligned} m(A, B) &= \frac{(3a-7)-(3b-7)}{a-b} \\ &= \frac{3a-7-3b-(-7)}{a-b} \\ &= \frac{3a-3b-7+7}{a-b} \\ &= \frac{3a-3b}{a-b} \\ &= \frac{3(a-b)}{a-b} \\ &= 3 \cdot \frac{a-b}{a-b} \\ &= 3 \end{aligned}$$

So no matter what a and b are, the slope between A and B will always be 3.

On Your Own

7. Answers may vary. Sample:

1. Test a few points.
2. Keep track of your steps!
3. Develop a point-tester.
4. Translate your point-tester into an equation.

8. (a) First, calculate the slope between $(5, 2)$ and $(-3, -4)$.

$$\frac{2-(-4)}{5-(-3)} = \frac{6}{8} = \frac{3}{4}$$

Next, write a point-tester equation. (x, y) will be on the line if the slope between (x, y) and $(5, 2)$ is $\frac{3}{4}$.

$$\begin{aligned} \frac{y-2}{x-5} &= \frac{3}{4} \\ y-2 &= \frac{3}{4}(x-5) \\ y-2 &= \frac{3}{4}x - \frac{15}{4} \\ y-2+2 &= \frac{3}{4}x - \frac{15}{4} + 2 \\ y &= \frac{3}{4}x - \frac{7}{4} \end{aligned}$$

(b) First, calculate the slope between $(5, 4)$ and $(0, 0)$.

$$\frac{4-0}{5-0} = \frac{4}{5}$$

Next, write a point-tester equation.

$$\begin{aligned} \frac{y-0}{x-0} &= \frac{4}{5} \\ y &= \frac{4}{5}x \end{aligned}$$

(c) Calculate the slope between $(5, 5)$ and $(7, 7)$:

$$\frac{7-5}{7-5} = \frac{2}{2} = 1$$

Write a point-tester equation.

$$\begin{aligned} \frac{y-5}{x-5} &= 1 \\ y-5 &= x-5 \\ y &= x \end{aligned}$$

(d) You already have the slope, $\frac{2}{3}$, so you can write a point-tester equation directly.

$$\begin{aligned} \frac{y-4}{x-5} &= \frac{2}{3} \\ y-4 &= \frac{2}{3}(x-5) \\ y-4 &= \frac{2}{3}x - \frac{10}{3} \\ y &= \frac{2}{3}x + \frac{2}{3} \end{aligned}$$

(e) Since the line is parallel to the line in part (a), the two lines will have the same slope, $\frac{3}{4}$. The point-tester equation is

$$\frac{y-0}{x-0}=\frac{3}{4}$$
$$y=\frac{3}{4}x$$

(f) The slope is $\frac{2}{3}$, so the point-tester equation is

$$\frac{y-0}{x-0}=\frac{2}{3}$$
$$y=\frac{2}{3}x$$

9. (a) The slope is $\frac{6-(-4)}{-5-(-3)}=\frac{10}{-2}=-5$.

(b) When a line is written in the form $y=ax+b$, the slope of the line is a. So the slope of this line is $\frac{3}{4}$.

(c) You can find the slope in two ways: either solve for y so the equation is in the form $y=ax+b$, in which case the slope will be a, or find two easy points (like the x- and y-intercepts) and calculate the slope between them. The two intercepts will be $(0,2)$ and $(4,0)$, so the slope is

$$\frac{0-2}{4-0}=\frac{-2}{4}=-\frac{1}{2}$$

(d) First, note that this equation and the one in part (c) are both written in the form $y=ax+b$, and that their values for a and b are the same. As in part (c), find the two intercepts and calculate the slope between them. The intercepts are $\left(\frac{15}{2},0\right)$ and $\left(0,\frac{15}{4}\right)$, and the slope between these points is

$$\frac{0-\frac{15}{4}}{\frac{15}{2}-0}=\frac{-\frac{15}{4}}{\frac{15}{2}}=\frac{-15}{4}\cdot\frac{2}{15}=\frac{-2}{4}=-\frac{1}{2}$$

(e) Finding two points on this line is not much fun. But if you solve the equation for y so it is in the form $y=ax+b$, the slope will be a, the coefficient of x.

$$5.1476x+5.1476y=15$$
$$5.1476y=-5.1476x+15$$
$$y=\frac{-5.1476}{5.1476}x+\frac{15}{5.1476}$$
$$y=-x+\frac{15}{5.1476}$$

Don't worry about simplifying the other messy fraction—you don't need it for slope. It's just -1.

(f) This equation is written in point-tester form. The slope is on the right side of the equation, $\frac{7}{13}$.

(g) This equation is also written in point-tester form, so the slope is also $\frac{7}{13}$. Since it has the same slope, this line is parallel to the line in part (f).

(h) The line will be parallel if it has the same slope as the line in part (d), $-\frac{1}{2}$.

10. The slope between J and K will be the same for both parts (a) and (b), so calculate it first:

$$m(J,K)=\frac{4-(-6)}{8-3}=\frac{10}{5}=2$$

(a) Your answers may vary, but a good place to start is with the slope point-tester. Using J as the base point, the point-tester would be

$$\frac{y+6}{x-3}=2$$

You can then multiply both sides by $(x-3)$ to get

$$y+6=2(x-3)$$

(b) Using point K as the base point, the point-tester would be

$$\frac{y-4}{x-8}=2$$

Multiply both sides by $(x-8)$ to get

$$y-4=2(x-8)$$

(c) There are a number of ways you can go about showing the two equations are equivalent. One way is to solve both equations for y, and simplify the right side as much as possible. That way, both equations should end up being the same if they are equivalent.

$$\begin{aligned} y+6&=2(x-3) \qquad & y-4&=2(x-8)\\ y&=2(x-3)-6 & y&=2(x-8)+4\\ y&=2x-6-6 & y&=2x-16+4\\ y&=2x-12 & y&=2x-12 \end{aligned}$$

So, since both equations ended up being equivalent to $y=2x-12$, they must be equivalent to each other.

11. (a) Each candy bar costs \$.85, so if Alejandra sells 20, she will make $20(\$.85)=17$ dollars.

(b) If she sells 50, she will make $50(\$.85)=\$42.50$.

(c) Look at the calculations you did for the number of dollars made (her sales, s). The number of candy bars sold c, was multiplied by 0.85. So the equation is

$$s=0.85(c)$$

The correct answer choice is **B**.

12. Find two points on the line, then calculate the slope between them. If $y=0$, then

$$5x+2(0)=10$$
$$5x=10$$
$$x=2$$

$(2,0)$ is one point on the line. To find another point, let $x=0$.

$$5(0)+2y=10$$
$$2y=10$$
$$y=5$$

$(0,5)$ is another point on the line. Now, calculate the slope between $(2,0)$ and $(0,5)$.

$$\frac{0-5}{2-0}=\frac{-5}{2}=-\frac{5}{2}$$

So **B**, $-\frac{5}{2}$, is the answer.

13. Use the point-tester method to find an equation. So a point (x, y) is on the line if it satisfies the equation

$$\frac{y-3}{x-5} = -4$$

Multiply both sides by $(x-5)$ to get

$$y - 3 = -4(x-5)$$

You cannot find a point that satisfies the equation but is not on the line. Remember, a point-tester is testing whether a point satisfies the equation—that is, that if you plug in the point, the equation is true. And if the point makes the point-tester true, then the point must be on the line.

14. You can answer this problem much in the same way as you tackled Exercise 6. First, find the slope of the line $ax + y = b$. You will not get an exact number, but an expression in terms of a and b.

Find two arbitrary points like you did in Exercise 6. Pick a letter for the x-coordinate and find an expression for the y-coordinate. You can't use a or b this time, since they're in the equation, so use r.

$$\begin{aligned} ax + y &= b \\ a(r) + y &= b \\ ar + y - ar &= b - ar \\ y &= b - ar \end{aligned}$$

So the point $(r, b - ar)$ is on the line (call it R). The second point, S, can be $(s, b - as)$. Now, calculate the slope.

$$\begin{aligned} m(R, S) &= \frac{(b-ar)-(b-as)}{r-s} \\ &= \frac{b - ar - b - (-as)}{r-s} \\ &= \frac{b - b - ar - (-as)}{r-s} \\ &= \frac{-ar - (-as)}{r-s} \\ &= \frac{-a(r-s)}{r-s} \\ &= -a \cdot \frac{r-s}{r-s} \\ &= -a \end{aligned}$$

So no matter what r and s are, the slope between R and S will always be $-a$.

You can convince yourself even further by solving the equation for y and finding the coefficient of x.

$$\begin{aligned} ax + y &= b \\ ax + y - ax &= -ax + b \\ y &= -ax + b \end{aligned}$$

The coefficient of x is $-a$, which is the same slope that you found the other way (good thing!).

15. Side $\overline{AB}$: Start by calculating $m(A, B)$:

$$\begin{aligned} m(A, B) &= \frac{2-7}{5-3} \\ &= -\frac{5}{2} \end{aligned}$$

Use the point-tester method, with slope $-\frac{5}{2}$ and base point $(5, 2)$:

$$\begin{aligned} \frac{y-2}{x-5} &= -\frac{5}{2} \\ y - 2 &= -\frac{5}{2}(x-5) \end{aligned}$$

Side $\overline{AC}$:

$$\begin{aligned} m(A, C) &= \frac{2-4}{5-10} \\ &= \frac{-2}{-5} \\ &= \frac{2}{5} \end{aligned}$$

The point-tester equation is

$$\begin{aligned} \frac{y-2}{x-5} &= \frac{2}{5} \\ y - 2 &= \frac{2}{5}(x-5) \end{aligned}$$

Side $\overline{BC}$:

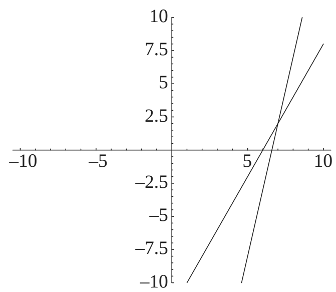
$$\begin{aligned} m(B, C) &= \frac{7-4}{3-10} \\ &= \frac{3}{-7} \\ &= -\frac{3}{7} \end{aligned}$$

The point-tester equation is

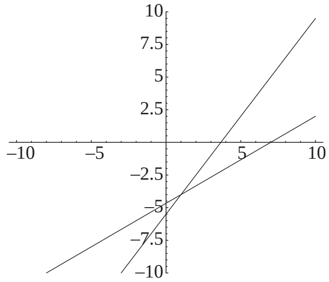
$$\begin{aligned} \frac{y-7}{x-3} &= -\frac{3}{7} \\ y - 7 &= \frac{3}{7}(x-3) \end{aligned}$$

Maintain Your Skills

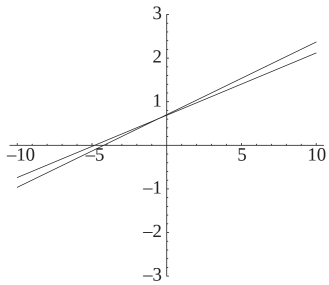
16. (a)



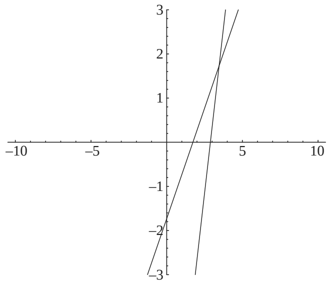
(b)



(c)

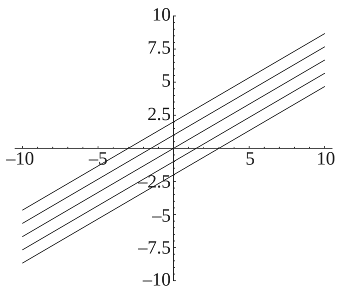


(d)



(e) All of the lines intersect. In fact, all of the lines are written in the form $(y - b) = m(x - a)$, and all lines written in the form will intersect at the point (a, b).

17. (a)



(b) Here are some conjectures you can make about this family:

- All members of the family have the same slope: $\frac{2}{3}$.
Justification: Pick any two points on the line $2x - 3y = k$. For example, you can use the x- and y- intercepts: $A = \left(\frac{k}{2}, 0\right)$ and $B = \left(0, \frac{k}{3}\right)$.

$$m(A, B) = \frac{0 - \left(-\frac{k}{3}\right)}{\frac{k}{2} - 0} = \frac{\frac{k}{3}}{\frac{k}{2}} = \frac{k}{3} \cdot \frac{2}{k} = \frac{2}{3}$$

So the slope does not depend on k.

- No two members have the same x-intercept and/or y-intercept unless the equations are equivalent.
Justification: As you figured above, the x-intercept is $x = \frac{k}{2}$ and the y-intercept is $y = -\frac{k}{3}$, which shows that both intercepts depend on the value of k; i.e., for each value of k there is a unique x-intercept and a unique y-intercept.
- The y-intercept is always $-\frac{k}{3}$.
Justification:
$$\begin{aligned} 2(0) - 3y &= k \\ -3y &= k \\ y &= -\frac{k}{3} \end{aligned}$$
- The x-intercept is always $\frac{k}{2}$.
Justification:
$$\begin{aligned} 2x - 3(0) &= k \\ 2x &= k \\ x &= \frac{k}{2} \end{aligned}$$
- There are an infinite number of members of this family.
Justification: For every value of k, there is a different equation that has different intercepts than every other equation in the family. Since there are an infinite number of k-values from which to choose, there are an infinite number of family members.

18. (a) Answers may vary. Samples are given;

$$\begin{array}{ll} y - 1 = 2(x - 5) & y = 4x - 19 \\ y - 1 = 10(x - 5) & y = -x + 4 \\ y - 1 = -\dfrac{3}{4}(x - 5) & y = 100x - 499 \\ y - 1 = 2x - 10 & y = 1 \\ y = 3(x - 5) + 1 & x = 5 \end{array}$$

(b) Here are some conjectures you can make about this family:

- Any equation in the form $y - 1 = m(x - 5)$ will go through the point $(5, 1)$.
Justification: Try plugging in the point $(5, 1)$ when m can be any value.
$$\begin{aligned} y - 1 &= m(x - 5) \\ (1) - 1 &\stackrel{?}{=} m((5) - 5) \\ 0 &\stackrel{?}{=} m(0) \\ 0 &= 0 \quad \checkmark \end{aligned}$$
More generally, the point (a, b) will always be on the graph of the line whose equation is $y - b = m(x - a)$.
- No two members of this family have the same slope unless their equations are equivalent.

Justification: It's tough to justify this conjecture directly. But focus on the form of the equation $y - 1 = m(x - 5)$. You must be able to write the equation of any line that goes through the point $(5, 1)$ in this form. (Why?) The line with this equation has slope m. If the two lines have the same slope, when you change them to this form, they will both end up with the same m—there is no way to vary the equation. The equations must then be equivalent.

- Members of this family do not intersect at any points other than $(5, 1)$.
  Justification: By the same reasoning as in the last conjecture, since two lines can only intersect in at most one point.
- There are an infinite number of members of this family.
  Justification: For every value of m, there is a different equation that has a different slope than every other equation in the family. Since there are an infinite number of m-values from which to choose, there are an infinite number of family members.

4.07 Jiffy Graphs: Lines

Check Your Understanding

1. The quickest way to write an equation depends on the information given. Each part in this exercise gives a point and a slope, so students can immediately write a point-tester.

(a) The point-tester would be

$$\frac{y-3}{x-4} = -1$$

Multiply each side by $(x - 4)$ and get

$$y - 3 = -(x - 4)$$

Students can simplify this equation further, but this answer is just fine.

(b) The point-tester is

$$\frac{y-5}{x-(-2)} = \frac{1}{3}$$

$x - (-2)$ is the same as $x + 2$, so multiply each side by $(x + 2)$ to get

$$y - 5 = \frac{1}{3}(x + 2)$$

(c) By now, you may see the pattern. Students do not have to start back at the point-tester, but instead can jump directly to the equation of the form

$$y - a = m(x - b)$$

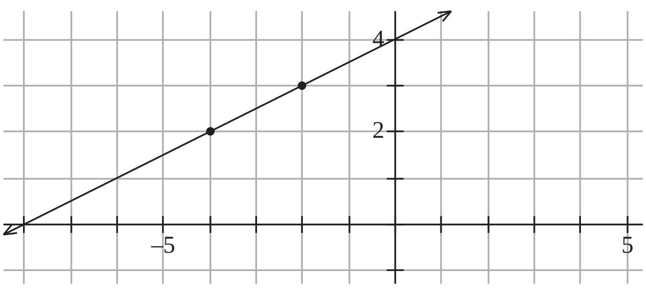
For this line, the equation is

$$y + 3 = -\frac{4}{5}(x - 7)$$

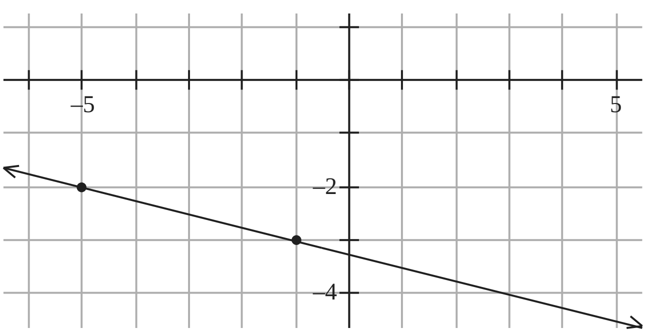
(d) The equation is $y + 5 = -\frac{11}{7}(x + 1)$.
(e) The equation is $y + \frac{1}{2} = 2\left(x + \frac{3}{2}\right)$.
(f) The equation is $y + 9.8 = -4.38(x - 14.6)$.
(g) The equation is $y - 12 = \frac{5}{6}x$.
(h) The equation is $y + 3 = -\frac{2}{3}x$.
(i) The equation is $y + \frac{8}{5} = \frac{5}{16}x$.
(j) The equation is $y - 5 = \frac{21}{13}x$.
(k) Follow the same process to get $y - 9 = 0(x - 3)$, which is fine, but leaving in that 0 makes the equation unnecessarily messy. Simplify the right side to get $y - 9 = 0$.
(l) Another tricky one, and you cannot use the point-tester. But, like part (k), you have seen problems like this one before. A line with no slope is vertical, and all points on a vertical line have the same x-coordinate, so the equation is $x = -8$.

2. The trick to this exercise is to recognize that all of the equations are in the form $y - b = m(x - a)$, or very close to it (the last three have the b on the other side). Based on work thus far, students know that the point (a, b) is on the graph of an equation of this form, and the slope of the line is m. So to make a sketch, find the point and use the slope to find a second point, then draw your line.

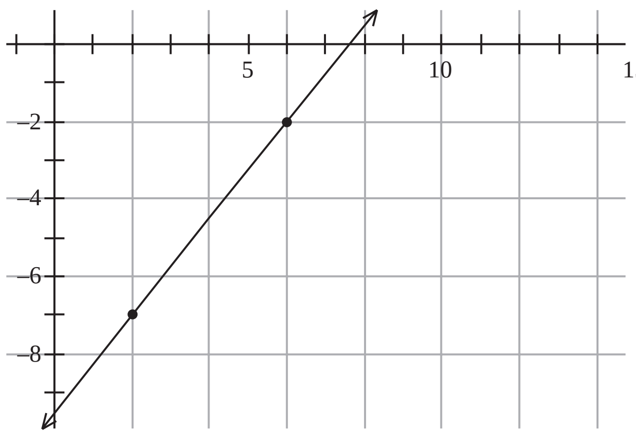
(a) One point on the graph is $(-4, 2)$. The slope is $\frac{1}{2}$, so to find another point, count up 1 and to the right 2 (since slope is "rise over run") and get the second point, $(-2, 3)$. Then connect the points.



(b) One point is $(-1, -3)$. The slope is -4, or $\frac{-4}{1}$, so rise -4 (that is, add -4 to the y-coordinate) and run 1 (add 1 to the x-coordinate) to get the second point, $(-5, -2)$.



(c)



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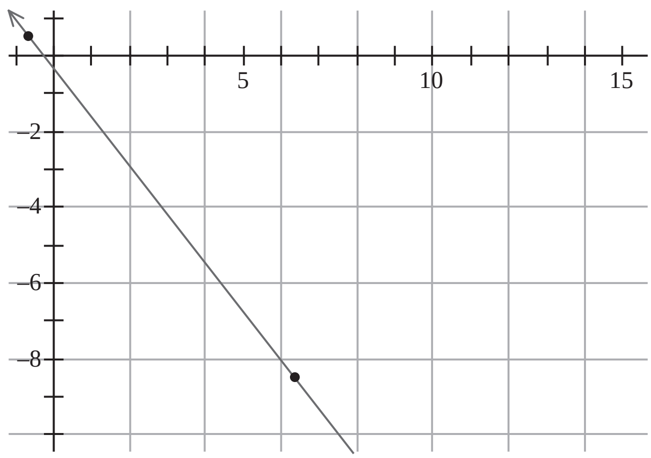
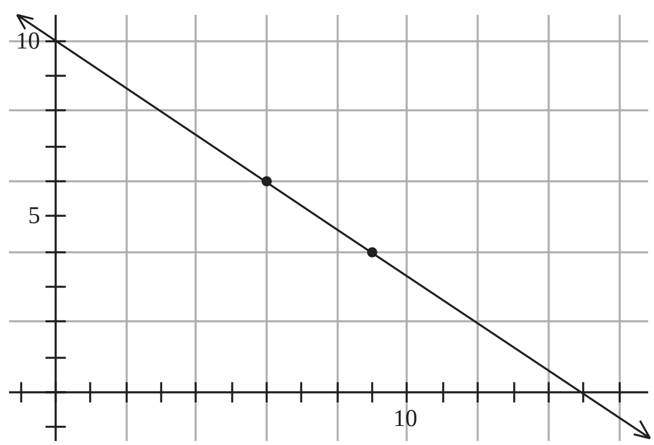


(e) First, subtract 6 from each side to get the form $y - 6 = -\frac{2}{3}(x - 6)$, so the point $(6, 6)$ is on the line, and the slope is $-\frac{2}{3}$.



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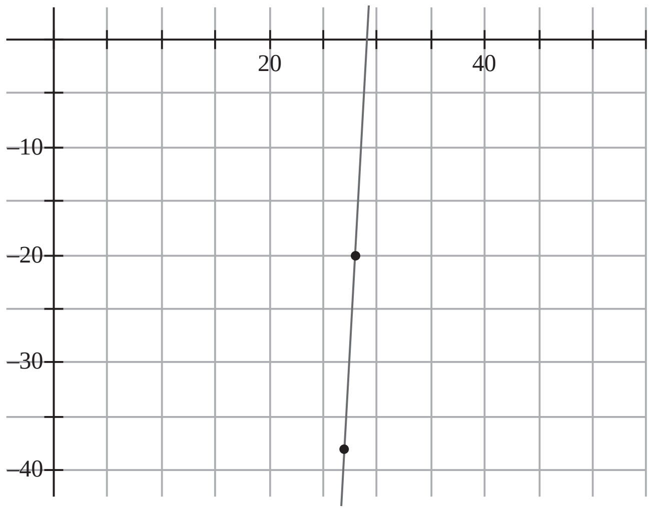

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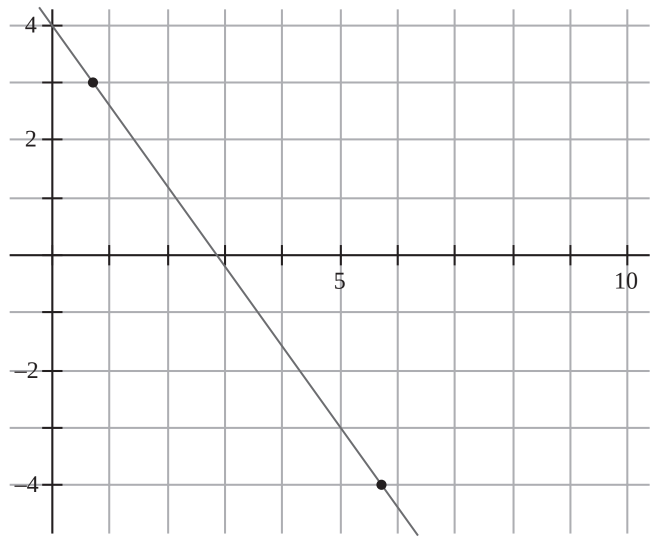

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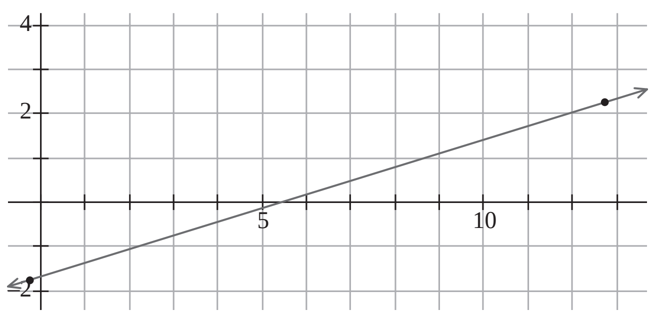


3. The form of all the equations in this exercise is slightly different from the form in Exercise 2. All of these equations are of the form $y = ax + b$. There are still some quick ways to find the information necessary to sketch the graphs.

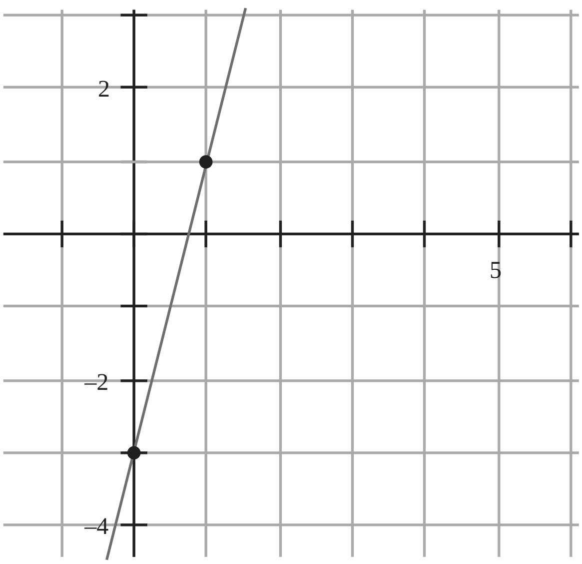
(a) First, find a point on the line. Since the equation is solved for y, it is usually easy to plug in 0 for x to get y. For this equation,

$$\begin{aligned} y &= 4x - 3 \\ y &= 4(0) - 3 \\ y &= -3 \end{aligned}$$

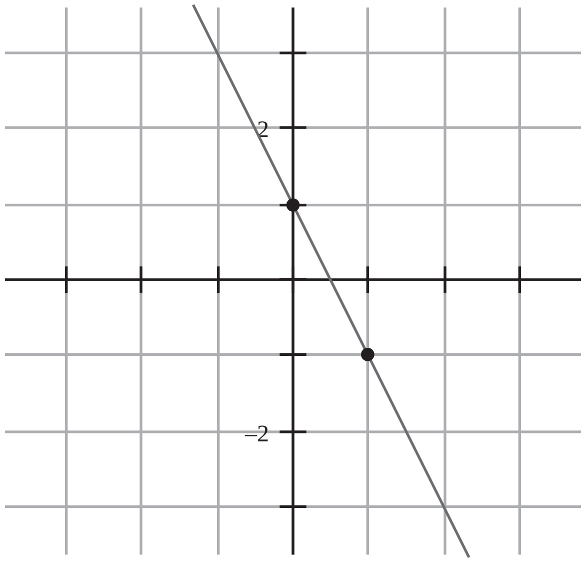
So the point $(0, -3)$ is on the line. (In fact, this point is where the line intersects the y-axis, since the x-coordinate is 0. This point is also called the y-intercept.) Now, find another point. Students can just plug in another value for x, like 1, and determine another y-value.

$$\begin{aligned} y &= 4x - 3 \\ y &= 4(1) - 3 \\ y &= 4 - 3 \\ y &= 1 \end{aligned}$$

So the second point is $(1, 1)$. With the two points, students can now draw a line.



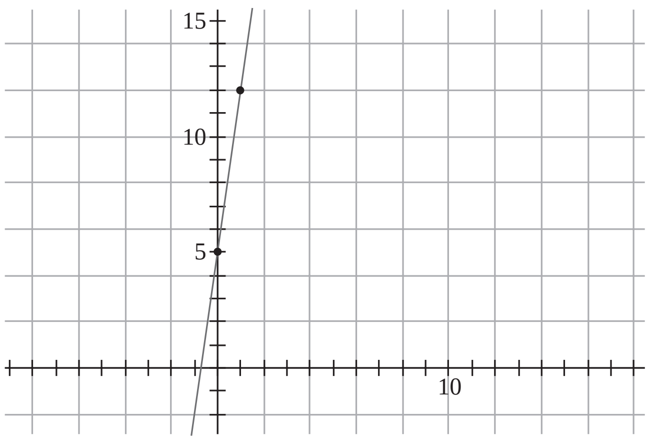
(b) If $x = 0$, then $y = 1$, so the point $(0, 1)$ is on the line. Notice that the b in the form $y = ax + b$ is the y-coordinate of the point where the line crosses the y-axis, that is, the y-intercept. Students can find one point from equations written this way. Again, find the second point by substituting 1 for x, which gives $y = -1$, so the point $(1, -1)$ is also on the line.



(c) The form $y = ax + b$ is very similar to the form $y - b = m(x - a)$. Notice what happens when you change this equation to that form.

$$\begin{aligned} y &= 7x + 5 \\ y - 5 &= 7x \\ y - 5 &= 7(x - 0) \end{aligned}$$

So the point $(0, 5)$ is on the line. Students also know that the slope of the line must be 7, since 7 is in the place of m in this form. That means that they can find the second point by going up 7 and to the right 1, giving the next point, $(1, 12)$.



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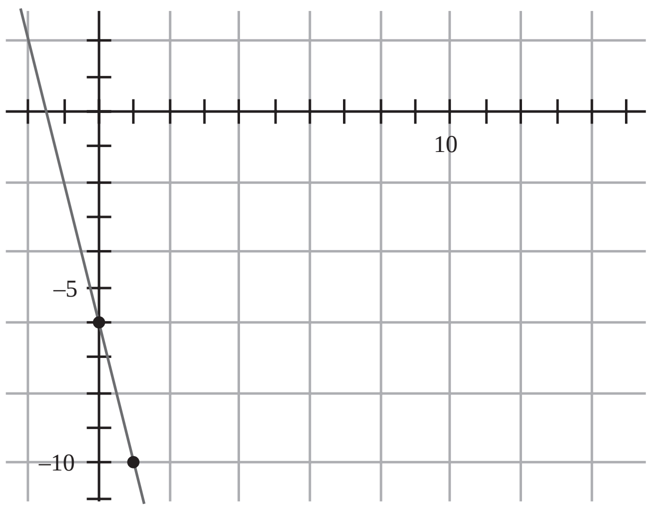
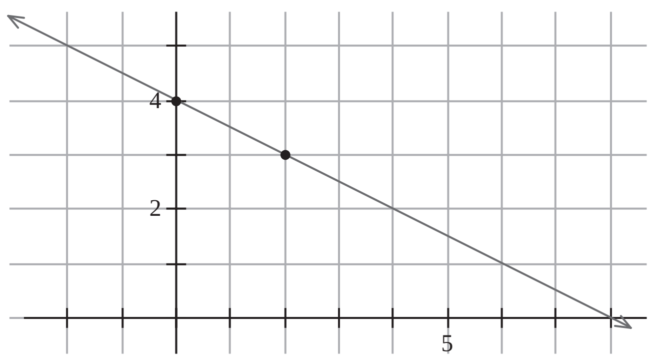


(e) The slope in this equation is a fraction. Students can use the rise-over-run method for finding a second point, but most students prefer to work with fractions as little as possible. Get rid of the fraction quickly by picking a convenient number for x, like 2.

$$y = -\frac{1}{2}x + 4$$
$$y = -\frac{1}{2}(2) + 4$$
$$y = -1 + 4$$
$$y = 3$$

So a second point is $(2, 3)$.



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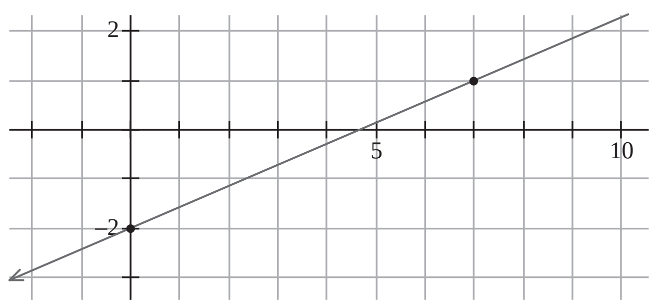

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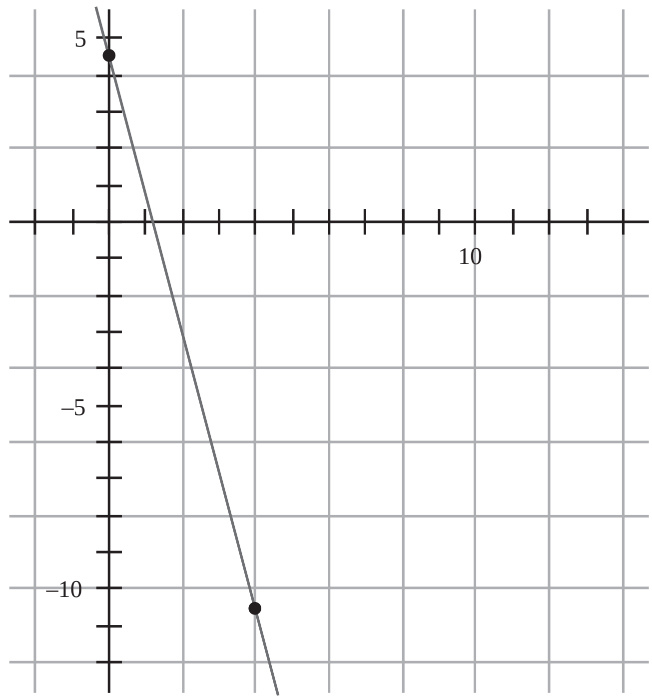

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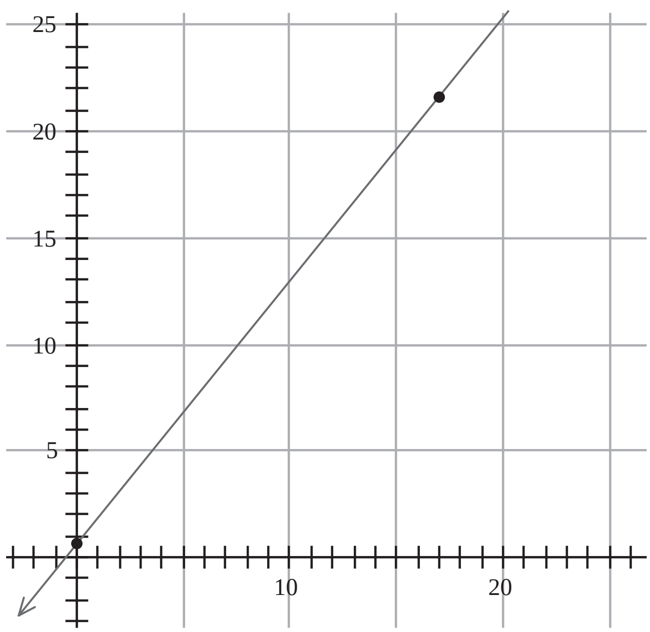


On Your Own

4. The value of k is found by replacing x with 100 in the final equation:

$$y = 4x - 9$$
$$y = 4(100) - 9$$
$$y = 391$$

The point $(100, 391)$ is on the graph. To check, see if the slope from it to $(3, 3)$ is 4:

$$\begin{aligned}\frac{y-3}{x-3} &= 4\\ \frac{391-3}{100-3} &\stackrel{?}{=} 4\\ \frac{388}{97} &= 4 \quad \checkmark\end{aligned}$$

5. Answers will vary. It does not matter which form of an equation students use to graph a line. As long as two equations are equivalent, their graphs will always be the same. Which form students prefer, of course, is up to them.

6. (a) The equation $m = 0.85(c) - 20$ is the only one that satisfies $c = 0$ and $m = -20$. Also, since each candy Alejandra sells earns \$0.85, and she starts at -20, the correct equation is **B**.

(b) She needs to earn \$20.00 by selling \$0.85 candy bars. So, she will have to sell $\frac{20}{0.85}$ to break even. Since $\frac{20}{0.85} \approx 23.5$, she will need to sell 24 candy bars. You can check the answer by trying out $c = 23$ and $c = 24$.

If $c = 23$, then she will have made $0.85(23) - 20 = -0.45$ So she will have made -0.45. Since -0.45 is negative, she has still lost some money.

On the other hand if $c = 24$, then she will have made $0.85(24) - 20 = 0.40$ So she will have made 0.40. Since 0.40 is positive, she will have made some money.

7. If the x-intercept is $(-a, 0)$ and the y-intercept is $(0, -b)$, then the slope is $\frac{-b-0}{0-(-a)} = \frac{-b}{a}$. So, the equation of the line in slope-intercept form is $y = \frac{-b}{a}x - b$. The correct answer is **D**.

To solve Exercises 8–27 quickly, first find the slope (unless it is already given), then pick one point and use the point-tester form to get your equation. Students do not have to simplify the equation unless asked to (in which case, final answers may not match the ones here).

8. The slope is

$$\frac{11-(-5)}{1-(-3)} = \frac{16}{4} = 4$$

So an equation would be

$$y + 5 = 4(x + 3)$$

9. The slope is already given to you, so just plug in values in the form $y - b = m(x - a)$.

$$y + 7 = \frac{7}{5}(x - 6)$$

10. Slope: $\frac{13-(-23)}{-5-4} = \frac{36}{-9} = -4$
Equation: $y - 13 = -4(x + 5)$

11. $y - 13 = \frac{11}{3}x$

12. $y + \frac{3}{2} = -\frac{2}{3}\left(x + \frac{1}{2}\right)$

13. $y + 1 = 3(x - 14)$

14. Slope: $\frac{9-7}{-4-0} = \frac{2}{-4} = -\frac{1}{2}$
Equation: $y - 7 = -\frac{1}{2}x$

15. Both points have the same x-coordinate, so the line is vertical. The equation is then $x = 4$.

16. Slope: $\frac{0-(-15)}{-6-2} = \frac{15}{-8} = -\frac{15}{8}$
Equation: $y = -\frac{15}{8}(x + 6)$

17. $y + 2 = 5x$

18. Slope: $\frac{11-0}{0-(-3)} = \frac{11}{3}$
Equation: $y = \frac{11}{3}(x + 3)$
Notice that the two points are intercepts. Look what happens when you write the equation in standard form:

$$\begin{aligned} y &= \frac{11}{3}(x+3) && \text{Multiply both sides by 3.}\\ 3y &= 11(x+3) && \text{Distribute the 11.}\\ 3y &= 11x + 33 && \text{Add } -11x \text{ to both sides.}\\ -11x + 3y &= 33 && \text{Multiply both sides by } -1.\\ 11x - 3y &= -33 \end{aligned}$$

Notice that the coefficient of x is the same as the y-coordinate of the point where the line crosses the y-axis (the y-intercept), and the coefficient of y is the x-intercept. The other side of the equation is the *product* of the x- and y-intercepts. Look what happens with arbitrary points, $(b, 0)$ and $(0, a)$:

The slope will be $\frac{a-0}{0-b} = -\frac{a}{b}$.

So the equation will be $y - a = -\frac{a}{b}x$. When you change the equation to the form $ax + by = c$, you get

$$\begin{aligned} y - a &= -\frac{a}{b}x && \text{Multiply both sides by } b.\\ by - ab &= -ax && \text{Add } ab \text{ to both sides.}\\ by &= -ax + ab && \text{Add } ax \text{ to both sides.}\\ ax + by &= ab \end{aligned}$$

So another quick way to write an equation when you have the two intercepts is to use the form $ax + by = ab$, where a is the y-coordinate of the y-intercept, and b is the x-coordinate of the x-intercept.

19. $y + \frac{1}{2} = \frac{5}{4}x$

20. $y + 4 = 3\frac{1}{3}(x + 2)$ is fine, but you usually do not use mixed fractions in equations, since they can get confusing. So change the slope from $3\frac{1}{3}$ to improper form, $\frac{10}{3}$.

$$y + 4 = \frac{10}{3}(x + 2)$$

20. Slope: $\frac{-2-(-8)}{8-2} = \frac{6}{6} = 1$
Equation: $y + 2 = 1(x - 8)$ or just $y + 2 = x - 8$

22. Unlike mixed fractions, decimals are just fine, since they're unlikely to cause confusion.

$$y - 3.6 = -4.8(x + 7.5)$$

23. $y - 3 = -\frac{7}{6}(x + 5)$

24. You can use the method described in the solution for Exercise 17 and jump to the equation

$$2x - 4y = -8$$

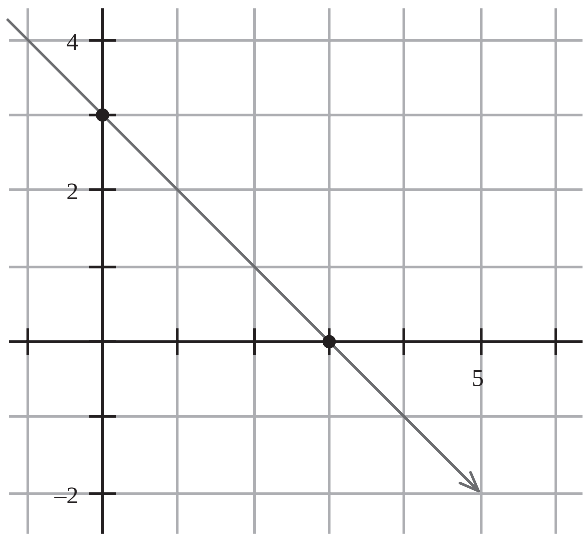
or you can find the slope and use the point-tester method.
Slope: $\frac{2-0}{0-(-4)} = \frac{2}{4} = \frac{1}{2}$
Equation: $y = \frac{1}{2}(x + 4)$

25. This line will go through the origin, so finding the slope is a little easier.
Slope: $\frac{-12-0}{15-0} = \frac{-12}{15} = -\frac{4}{5}$
Equation: $y - 0 = -\frac{4}{5}(x - 0)$, or simply $y = -\frac{4}{5}x$

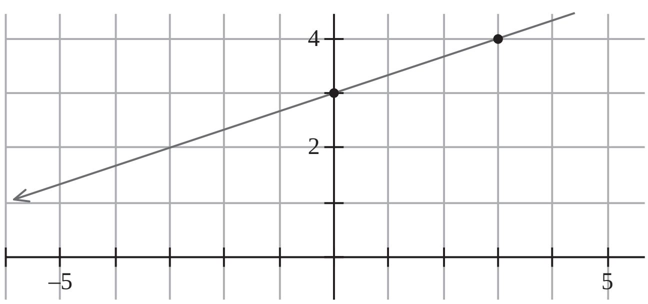
26. Slope: $\frac{0-(-1)}{5-(-3)} = \frac{1}{8}$
Equation: $y = \frac{1}{8}(x - 5)$

27. $y + 3 = 7(x - 10)$
To sketch a graph like those in Exercises 28–39, you need only find two points and draw a line that connects them. How you go about finding those two points can depend on the form of the equation you're given.

28. Two easy points to find are $(3, 0)$ and $(0, 3)$. Plot those points and draw the line.

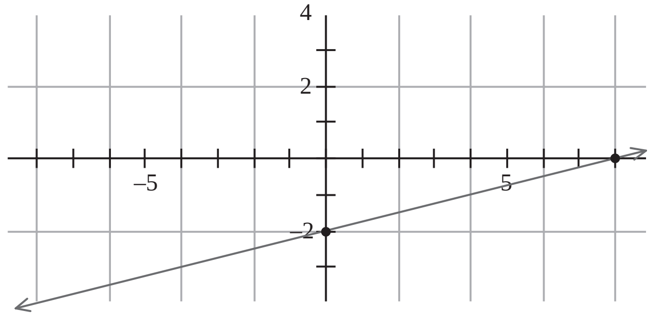


29. You can find one point quickly by substituting 0 for x, which gives you $y = 3$, so $(0, 3)$ is on the line. You can get a second point by substituting 3 for x, which gives you $y = \frac{1}{3}(3) + 3 = 1 + 3 = 4$, so the point $(3, 4)$ is also on the line.

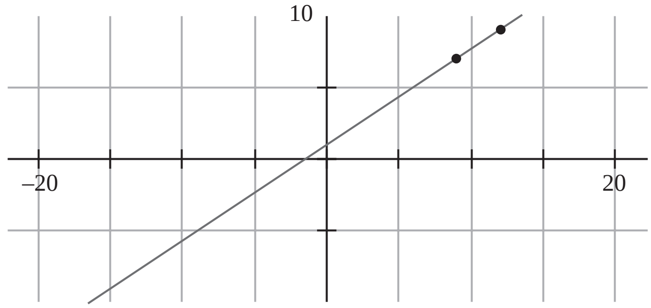


You may also notice that the equation is in the form $y = ax + b$. From earlier exercises, you may have found that the a in that form is the slope (for this line, that would be $\frac{1}{3}$), and the b in that form is the y-coordinate of the y-intercept (that is, the point $(0, 3)$ is on the line).

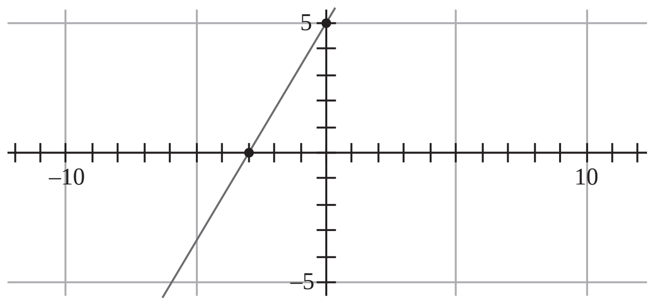
30. Again, quick points to find would be when $x = 0$ and $y = 0$. When $x = 0$, $y = -2$, and when $y = 0$, $x = 8$. So the two points are $(0, -2)$ and $(8, 0)$.



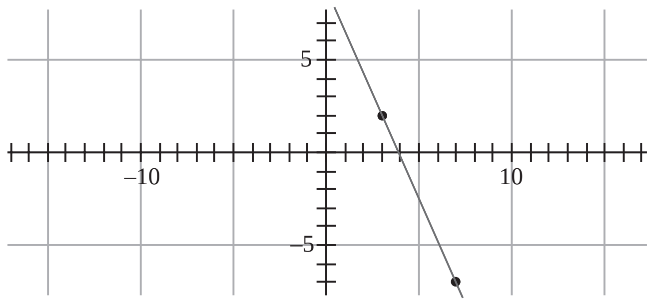
31. Since this equation is in the modified point-tester form, you can pick one point quickly, $(9, 7)$, and use the slope to find a second, $(9 + 3, 7 + 2) = (12, 9)$.



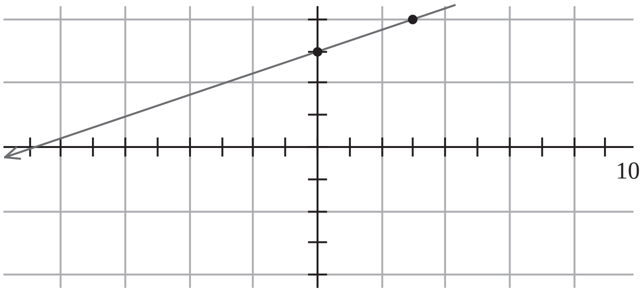
32. When $x = 0$, $y = 5$, and when $y = 0$, $x = -3$, so the points $(0, 5)$ and $(-3, 0)$ are on the graph.



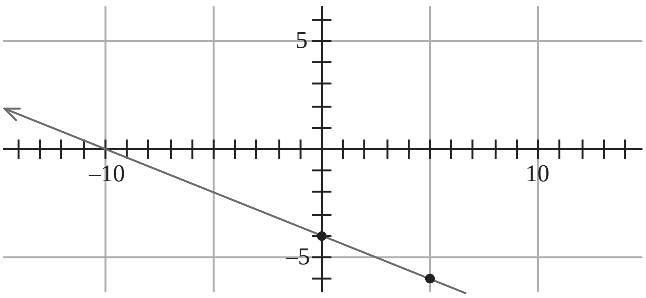
33. $(3, 2)$ is one point on the line, and a second is $(3 + 4, 2 - 9) = (7, -7)$.



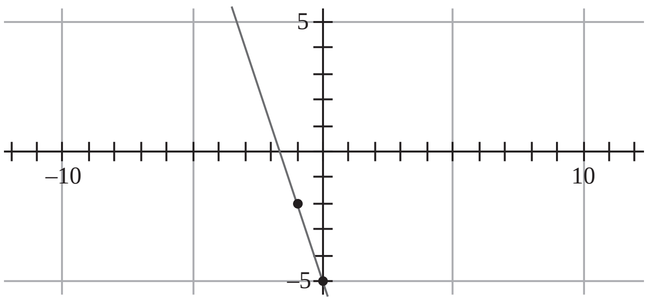
34. $(0, 3)$ is one point on the line, and a second is $(0+3, 3+1) = (3, 4)$.



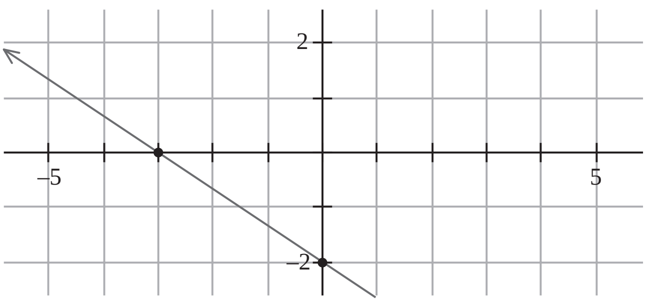
35. $(0, -4)$ is one point, a second is $(0+5, -4-2) = (5, -6)$.



36. $(-1, -2)$ is one point, $(-1+1, -2-3) = (0, -5)$ is another.

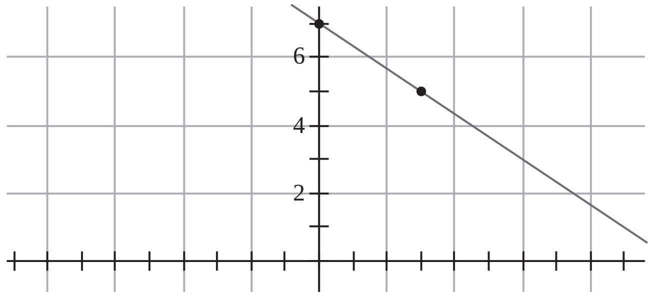


37. Two points are $(0, -2)$ and $(-3, 0)$.



38. Be careful with this one. You cannot use any of the shortcut tricks since it is not in any of the forms you need. You can still try points like before, or change it into a familiar form first.

If $x = 0$, then $-y = -7$, so $y = 7$. That puts $(0, 7)$ on the line. You can find a second point by substituting 3 for x to get $-y = \frac{2}{3}(3) - 7 = 2 - 7 = -5$, so if $-y = -5$, then $y = 5$.



You can also just multiply both sides by -1 to get the equation in a familiar form:

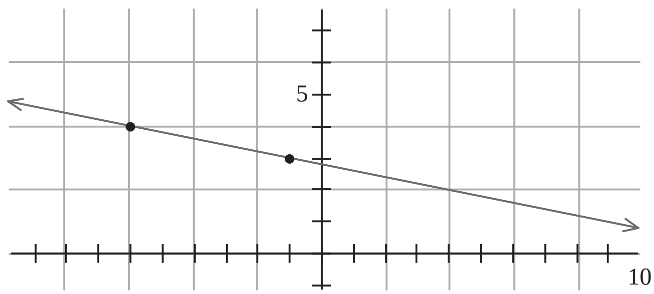
$$
\begin{aligned}
-y &= \frac{2}{3}x - 7 \\
(-1)(-y) &= (-1)\left(\frac{2}{3}x - 7\right) \\
y &= -\frac{2}{3}x + 7
\end{aligned}
$$

From this form, you can find the same two points, $(0, 7)$ and $(3, 5)$.

39. This equation is close to the modified point-tester form, except the left side is multiplied by 2. So if you multiply both sides by $\frac{1}{2}$, you will get the familiar form.

$$
\begin{aligned}
2(y-4) &= -\frac{2}{5}(x+6) \\
\frac{1}{2}(2(y-4)) &= \frac{1}{2}\left(-\frac{2}{5}(x+6)\right) \\
y - 4 &= -\frac{1}{5}(x+6)
\end{aligned}
$$

From this form, you can get the point $(-6, 4)$, and use the slope to get the second point, $(-6+5, 4-1) = (-1, 3)$.



For Exercises 40–49, use the basic rules and moves to convert each equation so that the left side is y, and the right has no y.

40.

$$
\begin{aligned}
-6x + y &= -1 && \text{Add } 6x \text{ to each side.} \\
6x - 6x + y &= 6x - 1 && \text{Simplify the left side.} \\
y &= 6x - 1
\end{aligned}
$$

41.

$$
\begin{aligned}
9x + y &= 4 && \text{Add } -9x \text{ to each side.} \\
-9x + 9x + y &= -9x + 4 && \text{Simplify the left side.} \\
y &= -9x + 4
\end{aligned}
$$

42.

$$\begin{aligned} 2x - 3y &= 15 && \text{Add } -2x \text{ to each side.} \\ -2x + 2x - 3y &= -2x + 15 && \text{Simplify the left side.} \\ 3y &= -2x + 15 && \text{Divide each side by 3.} \\ y &= -\frac{2}{3}x + 5 \end{aligned}$$

43.

$$\begin{aligned} -x - 4y &= -20 && \text{Add } x \text{ to each side.} \\ x - x - 4y &= x - 20 && \text{Simplify the left side.} \\ -4y &= x - 20 && \text{Divide each side by } -4. \\ y &= -\frac{1}{4}x + 5 \end{aligned}$$

44.

$$\begin{aligned} 3y + 7 &= 4x - 7y + 9 && \text{Add } 7y \text{ to each side.} \\ 3y + 7 + 7y &= 4x - 7y + 9 + 7y && \text{Simplify each side.} \\ 10y + 7 &= 4x + 9 && \text{Add } -7 \text{ to each side.} \\ 10y &= 4x + 2 && \text{Divide each side by 10.} \\ y &= \frac{2}{5}x + \frac{1}{5} \end{aligned}$$

45. There are two ways to work through this problem. For lots of fraction work, solve it this way.

$$\begin{aligned} &\frac{1}{2}x + \frac{2}{3}y - \frac{11}{4} = \frac{1}{3}y - \frac{9}{2} - \frac{3}{8}x && \text{Add } -\tfrac{1}{2}x \text{ to each side.} \\ &-\frac{1}{2}x + \frac{1}{2}x + \frac{2}{3}y - \frac{11}{4} \\ &\quad = -\frac{1}{2}x + \frac{1}{3}y - \frac{9}{2} - \frac{3}{8}x && \text{Simplify each side.} \\ &\frac{2}{3}y - \frac{11}{4} = -\frac{7}{8}x + \frac{1}{3}y - \frac{9}{2} && \text{Add } -\tfrac{1}{3}y \text{ to each side.} \\ &\frac{2}{3}y - \frac{11}{4} - \frac{1}{3}y = -\frac{7}{8}x \\ &\quad + \frac{1}{3}y - \frac{9}{2} - \frac{1}{3}y && \text{Simplify each side.} \\ &\frac{1}{3}y - \frac{11}{4} = -\frac{7}{8}x - \frac{9}{2} && \text{Add } \tfrac{11}{4} \text{ to each side.} \\ &\frac{1}{3}y - \frac{11}{4} + \frac{11}{4} = -\frac{7}{8}x - \frac{9}{2} + \frac{11}{4} && \text{Simplify each side.} \\ &\frac{1}{3}y = -\frac{7}{8}x - \frac{7}{4} && \text{Multiply each side by 3.} \\ &y = -\frac{21}{8} + \frac{21}{4} \end{aligned}$$

You can also start off by clearing all the fractions. Multiply each side by the least common denominator of all the fractions in the original equation, which is 24.

$$\begin{aligned} \frac{1}{2}x + \frac{2}{3}y - \frac{11}{4} &= \frac{1}{3}y - \frac{9}{2} - \frac{3}{8}x && \text{Multiply each side by 24.} \\ 12x + 16y - 66 &= 8y - 108 - 9x && \text{Add } -8y \text{ to each side.} \\ 12x + 8y - 66 &= -108 - 9x && \text{Add 66 to each side.} \\ 12x + 8y &= -42 - 9x && \text{Add } -12x \text{ to each side.} \\ 8y &= -21x - 42 && \text{Multiply each side by } \tfrac{1}{8}. \\ y &= -\frac{21}{8}x - \frac{21}{4} \end{aligned}$$

46.

$$\begin{aligned} 4x - 3(y - 4) + 6 &= 2x - y && \text{Distribute the } -3 \text{ to } y - 4. \\ 4x - 3y + 12 + 6 &= 2x - y && \text{Combine the } 12 + 6. \\ 4x - 3y + 18 &= 2x - y && \text{Add } -4x \text{ to each side.} \\ -3y + 18 &= -2x - y && \text{Add } y \text{ to each side.} \\ -2y + 18 &= -2x && \text{Add } -18 \text{ to each side.} \\ -2y &= -2x - 18 && \text{Multiply each side by } -\tfrac{1}{2}. \\ y &= x + 9 \end{aligned}$$

47.

$$\begin{aligned} y - (x - y) &= 5 && \text{Distribute the } -1 \text{ to } x - y. \\ y - x + y &= 5 && \text{Simplify the left side.} \\ 2y - x &= 5 && \text{Add } x \text{ to each side.} \\ 2y &= x + 5 && \text{Multiply each side by } \tfrac{1}{2}. \\ y &= \frac{1}{2}x + \frac{5}{2} \end{aligned}$$

48.

$$\begin{aligned} 2(x + y) - 4(x + y) &= 8(x + 2) && \text{Distribute over} \\ &\quad - 8(y + 2) && \text{each parentheses.} \\ 2x + 2y - 4x - 4y &= 8x + 16 && \text{Simplify each side.} \\ &\quad - 8y - 16 \\ -2x - 2y &= 8x - 8y && \text{Add } 2x \text{ to each side.} \\ -2y &= 10x - 8y && \text{Add } 8y \text{ to each side.} \\ 6y &= 10x && \text{Multiply each side by } \tfrac{1}{6}. \\ y &= \frac{5}{3}x \end{aligned}$$

You can simplify a little bit more quickly, but it is tricky. Notice that, on the left side, you have 2 times $(x + y)$ and -4 times $(x + y)$. So if think of each $(x + y)$ as a variable, such as z, you can add them together to say

$$2(x + y) - 4(x + y) = (2 - 4)(x + y) = -2(x + y)$$

You can do something similar on the right side, but it is a little harder to see.

$$\begin{aligned} 8(x + 2) - 8(y + 2) &= 8((x + 2) - (y + 2)) \\ &= 8(x - y + 2 - 2) = 8(x - y) \end{aligned}$$

So you can start with the simpler equation

$$-2(x + y) = 8(x - y)$$

49.

$$\begin{aligned} 2x + 3y - 4(x + 2y) &= 13 - 2(3y - x) + y && \text{Distribute the} \\ & && -4 \text{ and } -2. \\ 2x + 3y - 4x - 8y &= 13 - 6y && \text{Simplify each side.} \\ &\quad + 2x + y \\ -2x - 5y &= 13 - 5y + 2x && \text{Add } 5y \text{ to each side.} \\ -2x - 5y + 5y &= 13 - 5y && \text{Simplify each side.} \\ &\quad + 2x + 5y \\ -2x &= 13 + 2x \end{aligned}$$

There is no y left on either side, so you cannot solve this equation for y.

Maintain Your Skills

50. (a) First, as usual, find the slope between the two points.

$$\frac{5-0}{0-4} = -\frac{5}{4}$$

Then, use the point-tester to write an equation.

$$y = -\frac{5}{4}(x-4)$$

Finally, use the basic rules and moves to write the equation in the form $ax + by = c$.

$$\begin{aligned} y &= -\frac{5}{4}(x-4) && \text{Multiply each side by 4.} \\ 4y &= -5(x-4) && \text{Distribute the 5.} \\ 4y &= -5x + 20 && \text{Add } 5x \text{ to each side.} \\ 5x + 4y &= 20 \end{aligned}$$

(b) The slope between the two points is

$$\frac{0-(-2)}{1-0} = \frac{2}{1} = 2$$

The point-tester equation is

$$\begin{aligned} y &= 2(x-1) && \text{Distribute the 2.} \\ y &= 2x - 2 && \text{Add } -2x \text{ to each side.} \\ -2x + y &= -2 \end{aligned}$$

(c) The slope between the two points is

$$\frac{0-(-7)}{-15-0} = \frac{7}{-15} = -\frac{7}{15}$$

The point-tester equation is

$$\begin{aligned} y &= -\frac{7}{15}x + 15 && \text{Multiply each side by 15.} \\ 15y &= -7(x+15) && \text{Distribute the } -7. \\ 15y &= -7x - 105 && \text{Add } 7x \text{ to each side.} \\ 7x + 15y &= -105 \end{aligned}$$

(d) The slope between the two points is

$$\frac{4-0}{0-(-6)} = \frac{4}{6} = \frac{2}{3}$$

The point-tester equation is

$$\begin{aligned} y &= \frac{2}{3}(x+6) && \text{Multiply each side by 3.} \\ 3y &= 2(x+6) && \text{Distribute the 2.} \\ 3y &= 2x + 12 && \text{Add } -2x \text{ to each side.} \\ -2x + 3y &= 12 \end{aligned}$$

(e) As you might have seen in Exercise 18, you can find the equation of a line when given the x- and y-intercepts pretty quickly. When you wrote the equations in the form $ax + by = c$ (sometimes called *standard form*), the coefficient of x is the same as the y-coordinate of the point where the line crosses the y-axis (the y-intercept), and the coefficient of y is the x-coordinate of the x-intercept. The other side of the equation, the *constant* (c, the number that is not multiplied by a variable) is the product of the two intercepts.

- In the first equation, the points were $(4, 0)$ and $(0, 5)$. The equation in standard form is $5x + 4y = 20$.
- In the second equation, the points were $(1, 0)$ and $(0, -2)$. The equation in standard form is $-2x + y = -2$. You may have gotten something like $2x - y = 2$, but notice that these two equations are equivalent—just multiply each side of one by -1 to get the other.
- The third equation was from points $(-15, 0)$ and $(0, -7)$. The equation in standard form is $-7x - 15y = -105$, which is equivalent to $7x + 15y = 105$ if you multiply each side by -1.
- The fourth equation was from points $(-6, 0)$ and $(0, 4)$. The equation in standard form is $4x - 6y = -24$, which is equivalent to $-2x + 3y = 12$ if you multiply each side by $-\frac{1}{2}$.

Look what happens with arbitrary points, $(b, 0)$ and $(0, a)$:
The slope will be $\frac{a-0}{0-b} = -\frac{a}{b}$.
So the equation will be $y - a = -\frac{a}{b}x$. When you change the equation to the form $ax + by = c$, you get

$$\begin{aligned} y - a &= -\frac{a}{b}x && \text{Multiply each side by } b. \\ by - ab &= -ax && \text{Add } ab \text{ to each side.} \\ by &= -ax + ab && \text{Add } ax \text{ to each side.} \\ ax + by &= ab \end{aligned}$$

So another quick way to write an equation when you have the two intercepts is to use the form $ax + by = ab$, where a is the y-coordinate of the y-intercept, and b is the x-coordinate of the x-intercept.

51. (a) First, as usual, find the slope between the two points.

$$\frac{0-\left(-\frac{2}{3}\right)}{\frac{1}{2}-0} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{2}{3} \cdot \frac{2}{1} = \frac{4}{3}$$

Then, use the point-tester to write an equation.

$$y = \frac{4}{3}\left(x - \frac{1}{2}\right)$$

Finally, use the basic rules and moves to write the equation in the form $ax + by = c$.

$$\begin{aligned} y &= \frac{4}{3}\left(x - \frac{1}{2}\right) && \text{Multiply each side by 3.} \\ 3y &= 4\left(x - \frac{1}{2}\right) && \text{Distribute the 4.} \\ 3y &= 4x - 2 && \text{Add } -4x \text{ to each side.} \\ -4x + 3y &= -2 \end{aligned}$$

You can try to use the results of Exercise 50 too. If a is $-\frac{2}{3}$ and b is $\frac{1}{2}$, plug them into the form

$ax + by = ab$:

$$-\frac{2}{3}x + \frac{1}{2}y = \left(-\frac{2}{3}\right)\left(\frac{1}{2}\right) \quad \text{Simplify the right side.}$$
$$-\frac{2}{3}x + \frac{1}{2}y = -\frac{1}{3} \quad \text{Multiply each side by 6.}$$
$$-4x + 3y = -2$$

And you get the same equation.

(b) Use the method from Exercise 50 again, since it requires less work.

$$\frac{17}{4}x - \frac{11}{7}y = \left(\frac{17}{4}\right)\left(\frac{11}{7}\right) \quad \text{Simplify the right side.}$$
$$\frac{17}{4}x - \frac{11}{7}y = \frac{187}{28} \quad \text{Multiply each side by 28.}$$
$$119x - 44y = 187$$

(c)

$$\frac{10}{9}x + \frac{5}{9}y = \left(\frac{10}{9}\right)\left(\frac{5}{9}\right) \quad \text{Simplify the right side.}$$
$$\frac{10}{9}x + \frac{5}{9}y = \frac{50}{81} \quad \text{Multiply each side by 81.}$$
$$90x + 45y = 50 \quad \text{You can simplify a little more.}$$
$$18x + 9y = 10$$

(d)

$$\frac{1}{2}x - \frac{3}{8}y = \left(\frac{1}{2}\right)\left(-\frac{3}{8}\right) \quad \text{Simplify the right side.}$$
$$\frac{1}{2}x - \frac{3}{8}y = -\frac{3}{16} \quad \text{Multiply each side by 16.}$$
$$8x - 6y = -3$$

(e) You can use the same process as explored in Exercise 50, but you cannot immediately see the intercepts from the equation in standard form. In fact, the trick going the other way only works if in fact the constant is the product of the two coefficients.

There is a pattern, but it can be difficult to see. Take a look at the first pair of points, $\left(\frac{1}{2}, 0\right)$ and $\left(0, -\frac{2}{3}\right)$. The resulting equation was $-4x + 3y = -2$. The coefficient of x is the product of the numerator of the y-coordinate of the y-intercept and the denominator of the x-coordinate of the x-intercept. The coefficient of y is the product of the *denominator* of the y-coordinate of the y-intercept and the *numerator* of the x-coordinate of the x-intercept. The constant is the product of the two *numerators*.

If the two intercepts are of the form $\left(0, \frac{a}{b}\right)$ and $\left(\frac{c}{d}, 0\right)$, where a, b, c, and d are all integers, you can write a point-tester equation and simplify it.

$$\frac{a}{b}x + \frac{c}{d}y = \left(\frac{a}{b}\right)\left(\frac{c}{d}\right) \quad \text{Simplify the right side.}$$
$$\frac{a}{b}x + \frac{c}{d}y = \frac{ac}{bd} \quad \text{Multiply each side by } bd.$$
$$bd\left(\frac{a}{b}x\right) + bd\left(\frac{c}{d}y\right) = ac \quad \text{Simplify the right side.}$$
$$adx + bcy = ac$$

So, it might be tough to follow, but it does work.

52. (a) The first equation is $3x + y = 7$. If $x = 1$, then $y = 4$, since $3(1) + 4 = 7$. So the point $(1, 4)$ is on the graph. Find a second—when $x = 2$, then $y = 1$, since $3(2) + 1 = 7$. So the point $(2, 1)$ is also on the graph. Use the two points to find the slope.

$$\frac{4-1}{1-2} = \frac{3}{-1} = -3$$

(b) In Exercise 17 from Lesson 4.06 on page 357, you saw that if the equation is written in standard form, and the x- and y-coefficients are the same, then the lines are parallel. Plus, parallel lines have the same slope. So the slope of this line is also -3.

(c) Finding points on this line may be difficult, since there is a messy fraction for the constant. You can instead solve the equation for y and simplify. The coefficient of x will then be the slope.

$$3x + 7y = \frac{42}{13}$$
$$7y = -3x + \frac{42}{13}$$
$$y = -\frac{3}{7}x + \left(\frac{1}{7}\right)\left(\frac{42}{13}\right)$$

There is no need to multiply out that last fraction, since you're only after the slope. You can take it from the coefficient of x, making the slope $-\frac{3}{7}$.

(d) Solve for y to get $y = \frac{2}{7}x - \frac{9}{7}$, so the slope is $\frac{2}{7}$.

(e) The coefficients are the same as part (d), so the slope is also $\frac{2}{7}$

(f) Solve for y to get $y = \frac{5}{4}x - \frac{20}{4}$, so the slope is $\frac{5}{4}$.

(g) Solve for y to get $y = \frac{3}{4}x - \frac{15}{4}$, so the slope is $\frac{3}{4}$.

(h) Solve for y to get $y = \frac{1}{2}x - \frac{19}{4}$, so the slope is $\frac{1}{2}$.

(i) The numerator of the slope is the opposite of the x-coefficient, and the denominator is the y-coefficient. Note that sometimes the slope forms a fraction that can be reduced, as in the case of part (h).

4.08 Overtaking

Check Your Understanding

1. (a) Yakov, because his graph is steeper.

(b) Yakov overtakes Demitri at 2 seconds, the x coordinate of the point at which the two lines intersect.

(c) The distance is the y-coordinate of the point of intersection, which is 40 feet.

(d) Yakov's speed is 20 feet per second, and Demitri's speed is 10 feet per second. Speed, the rate of change of distance, is rise over run. Yakov travels 120 feet in 6 seconds, for a speed of $\frac{120}{6} = 20$. Demitri has a 20-foot head start, so he travels 100 feet in 10 seconds, for a speed of $\frac{100}{10} = 10$.

(e) Yakov's equation is $y = 20x$.
(f) Demitri's equation is $y = 10x + 20$.
(g) The point at which Yakov overtakes Demitri is the point at which they are at the same distance from the starting point. That is, Yakov's y value equals Demitri's y value:

$$\begin{aligned} 20x &= 10x + 20 \\ 20x - 10x &= 10x + 20 - 10x \\ 10x &= 20 \\ \frac{10x}{10} &= \frac{20}{10} \\ x &= 2 \end{aligned}$$

2. (a) Yakov and Demitri are going at the same speed, because neither line is steeper.
(b) Yakov doesn't overtake Demitri at all, because the graphs have no point of intersection.
(c) Both speeds are 20 feet per second.
(d) Yakov's equation is $y = 20x$.
(e) Demitri's equation is $y = 20x + 20$.
(f) The point at which Yakov overtakes Demitri is the point at which they are at the same distance from the starting point. That is, Yakov's y value equals Demitri's y value:

$$\begin{aligned} 20x &= 20x + 20 \\ 20x - 20x &= 20x + 20 - 20x \\ 0 &= 20 \end{aligned}$$

There is no solution to the equation, which suggests that the two runners will never meet.

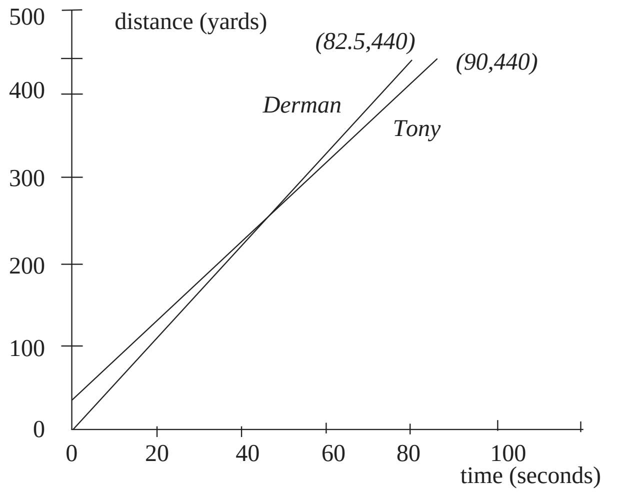
3. (a) Yakov doesn't overtake Demitri. Yakov is going faster than Demitri, and he had a head start.
(b) Yes, but not at a point that's meaningful for the race. The intersection takes place before the race begins.
(c) They would intersect at $(-2, -20)$.
(d) Two lines cannot intersect in more than one point. If two lines contain two or more points in common, they must have all points in common—they are the same line.

On Your Own

4. Answers will vary, but Yakov is going faster than Demitri, so he will eventually catch him. He gains 1 foot per minute on Demitri, who has a 50-foot head start, so it will take $\frac{50 \text{ feet}}{1 \text{ ft/min}} = 50$ minutes to catch up to him.

5. Derman is faster, since he runs the race in less time than Tony.

6.



Derman starts from $(0, 0)$. He can run a quarter mile in $1\frac{3}{8}$ minutes. A quarter mile is 440 yds, and $1\frac{3}{8}$ minutes is 82.5 seconds, so $(82.5, 440)$ is on Derman's graph. The slope of Derman's graph is

$$\frac{440}{82.5} = \frac{16}{3}$$

The equation of Derman's graph is

$$\frac{y - 0}{x - 0} = \frac{16}{3}$$

which simplifies to $3y = 16x$.

Tony starts 20 yards ahead, so, in the race, his initial position (when time is 0) is 20, represented by $(0, 20)$ on the graph. $1\frac{1}{2}$ minutes is 90 seconds, at which time his position is the 420 yards he ran, plus his 20-yard head start. The point $(90, 440)$ is also on his graph. His rate is 440 yards in 90 seconds, so the slope of Tony's distance-time graph is

$$\frac{440}{90} = \frac{44}{9}$$

The equation of Tony's graph is

$$\frac{y - 20}{x - 0} = \frac{44}{9}$$

which simplifies to $9y - 180 = 44x$.

7. Yes, Derman will overtake Tony, because the graphs cross. It looks as if the intersection point is about $(43, 250)$, so Derman will pass Tony in about 43 seconds, about 250 yards out.

To find out exactly where Derman will overtake Tony, you need the location where the two lines intersect. So, look for a point that satisfies both equations.

$$\begin{aligned} &\text{Equation D:} & 3y &= 16x \\ &\text{Equation T:} & 9y - 180 &= 44x \end{aligned}$$

Try the Guess-Check-Generalize method. From the graph, the lines seem to cross when $x = 43$.

- In equation D, if $x = 43$,

$$3y = 16 \cdot 43 \quad \text{so}$$
$$y = \frac{16 \cdot 43}{3} = 229\frac{1}{3}$$

- In equation T, if $x = 43$,

$$9y - 180 = 44 \cdot 43 \quad \text{so}$$
$$y = \frac{44 \cdot 43 + 180}{9} = 230\frac{2}{9}$$

So, $x = 43$ is close but not right. If "$x = a$,"

$$\frac{16a}{3} \stackrel{?}{=} \frac{44a + 180}{9}$$

and then solve the equation

$$\frac{16a}{3} = \frac{44a + 180}{9}$$

Multiply each side by 9 and then use the basic moves:

$$\begin{aligned} 9\left(\frac{16a}{3}\right) &= 9\left(\frac{44a+180}{9}\right) \\ 48a &= 44a + 180 \\ 4a &= 180 \\ a &= 45 \end{aligned}$$

So, the lines cross when $x = 45$. You can check that when $x = 45$, each equation produces the same y, namely 240. So, Derman overtakes Tony after 45 seconds, when they are 240 yards out.

8. Derman takes 82.5 seconds to run 440 yards, and Tony takes 85.9 seconds to run 420 yards. These answers come from the endpoints of the graphs. Or you could have used the equations and substituted 440 for y, then solve for x.

9. Yes, you will eventually catch up to the other person. Every moment that passes, you are getting closer to the other person. If you walk for long enough, you will catch up to and pass the other person.

10. No, you will never catch up to the other person. Since the two of you are walking at precisely the same speed, you are not gaining any ground. After every minute, you are just as far behind the person as you were when you started.

11. Let t be time and d the distance from Dallas. Then the equation for Mr. Merrill's distance from Dallas is $d = 62.5t$. Because Mrs. Merrill is 100 miles away when Mr. Merrill leaves Dallas, the equation for Mrs. Merrill's distance from Dallas is $d = 50t + 100$. Set these two equations equal and solve for t.

$$\begin{aligned} 62.5t &= 50t + 100 \\ 62.5t - 50t &= 100 \\ 12.5t &= 100 \\ t &= \frac{100}{12.5} = 8 \end{aligned}$$

So, Mr. Merrill catches up with Mrs. Merrill after 8 hours. He is driving at 62.5 miles per hour, so he will catch her $8(62.5) = 500$ miles away from Dallas. The answer is **A**.

Maintain Your Skills

12. (a) If a point (a, b) is on both graphs, then $b = 2a$ and $b = 3a$, so $2a = 3a$ and $a = 0$. If $a = 0$, then $b = 0$.

(b) If a point (a, b) is on both graphs, then $b = 2a + 1$ and $b = 3a + 1$, so $2a + 1 = 3a + 1$ and $a = 0$. If $a = 0$, then $b = 1$.

(c) If a point (a, b) is on both graphs, then $b = 2a - 1$ and $b = 3a - 1$, so $2a - 1 = 3a - 1$ and $a = 0$. If $a = 0$, then $b = -1$.

(d) If a point (a, b) is on both graphs, then $b + 1 = 2a$ and $b + 1 = 3a$, so $2a = 3a$ and $a = 0$. If $a = 0$, then $b = -1$.

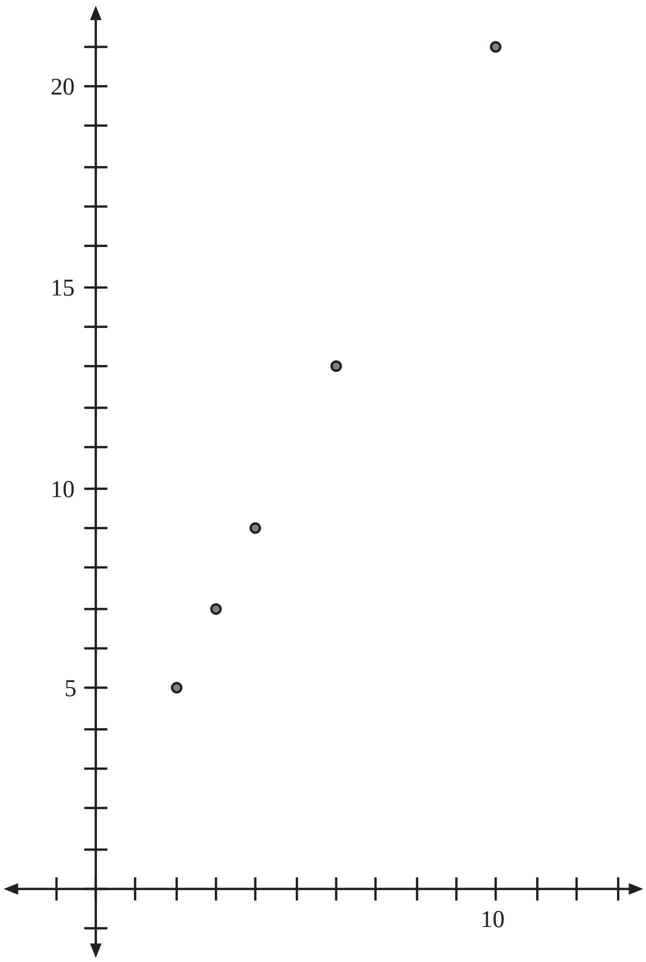
(e) If a point (a, b) is on both graphs, then $b - 2 = 2a$ and $b - 2 = 3a$, so $2a = 3a$ and $a = 0$. If $a = 0$, then $b = -1$.

(f) Each of these equations is equivalent to one in part (e), so the graphs will be the same and the intersection point will be the same.

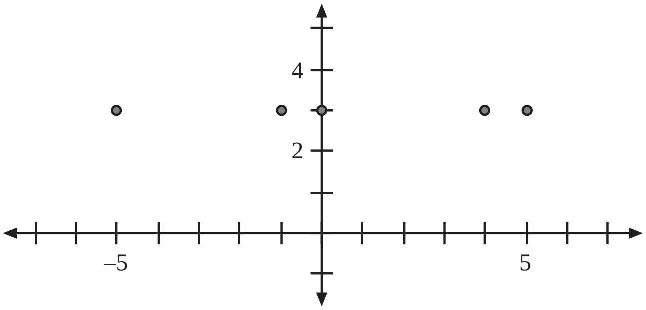
4B MATHEMATICAL REFLECTIONS

1. You can use the slope to find points that are collinear. Slope is $\frac{\text{rise}}{\text{run}}$. Just add or subtract the *rise* to the y-coordinate of a point and the *run* to the x-coordinate to find another point on the line.

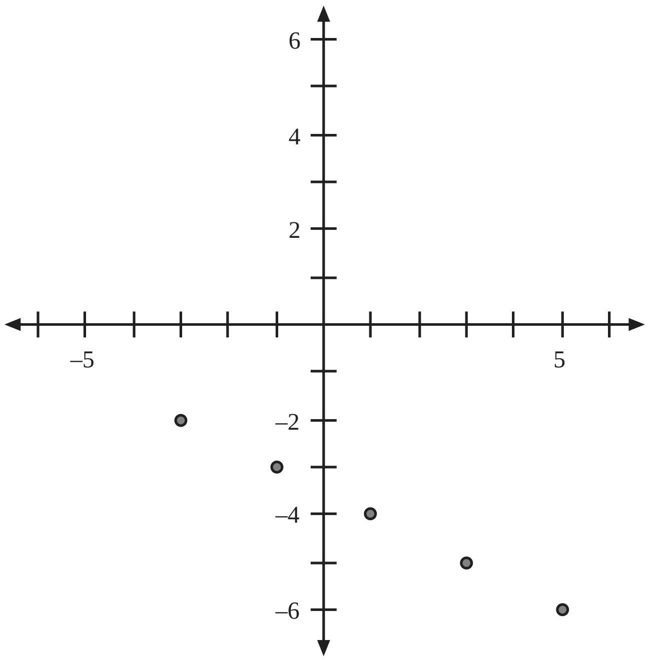
(a) Answers will vary. The slope is $\frac{13-5}{6-2} = \frac{8}{4} = \frac{2}{1}$. Possible answers: $(2+1, 5+2) = (3, 7)$, $(3+1, 7+2) = (4, 9)$, and $(6+4, 13+8) = (10, 21)$ $\left(\text{Remember, } \frac{2}{1} \text{ can be written in equivalent forms such as } \frac{4}{2} \text{ or } \frac{6}{3}\right)$.



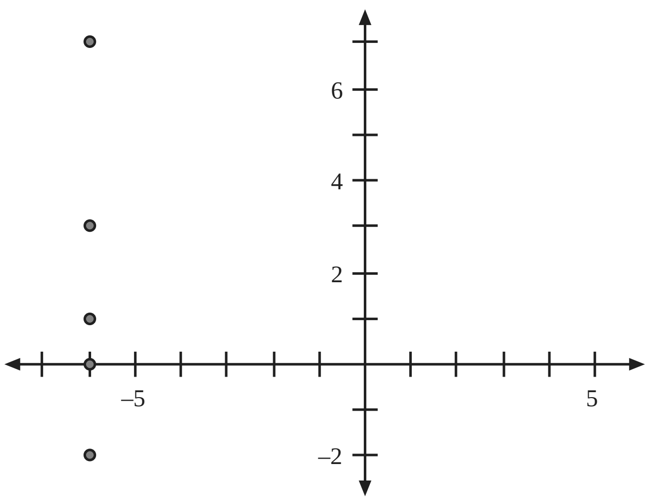
(b) Answers will vary. The slope is $\frac{3-3}{5-(-1)} = \frac{0}{6} = 0$. This is a horizontal line. All points will have the same y-coordinate, 3. Possible answers: $(0, 3)$, $(4, 3)$, and $(-5, 3)$



(c) Answers will vary. The slope is $\frac{-5-(-4)}{3-1} = \frac{-1}{2}$. Possible answers: $(1-2, -4-(-1)) = (-1, -3)$, $(-1-2, -3-(-1)) = (-3, -2)$, and $(3+2, -5+(-1)) = (5, -6)$



(d) Answers will vary. The slope is $\frac{0-7}{-6-(-6)} = \frac{-7}{0}$ which is undefined. The line is vertical. All points will have the same x-coordinate, -6. Possible answers: $(-6, 1)$, $(-6, 3)$, and $(-6, -2)$



2. (a) Using $A = (1, 4)$ as a base point, use a point-tester equation. $P = (x, y)$ is on the line if

$$\begin{aligned} m(P, A) &= \frac{1}{2} \\ \frac{y-4}{x-1} &= \frac{1}{2} \\ y - 4 &= \frac{1}{2}(x-1) \\ y - 4 &= \frac{1}{2}x - \frac{1}{2} \\ y &= \frac{1}{2}x - \frac{1}{2} + 4 \\ y &= \frac{1}{2}x + \frac{7}{2} \end{aligned}$$

(b) First find the slope. $m(Q, R) = \frac{2-(-4)}{0-1} = \frac{6}{-1} = -6$. Use either Q or R as a base point and use a point-tester equation. $P = (x, y)$ is on the line if

$$\begin{aligned} m(P, Q) &= -6 \\ \frac{y-(-4)}{x-1} &= -6 \\ y + 4 &= -6(x-1) \\ y + 4 &= -6x + 6 \\ y &= -6x + 6 - 4 \\ y &= -6x + 2 \end{aligned}$$

(c) If the slope is 0, the line is horizontal. All points will have the same y-coordinate. The equation is $y = 5$. The point-tester equation is

$$\begin{aligned} m(P, B) &= 0 \\ \frac{y-5}{x-(-2)} &= 0 \\ y - 5 &= 0(x+2) \\ y &= 0x + 5 \\ y &= 5 \end{aligned}$$

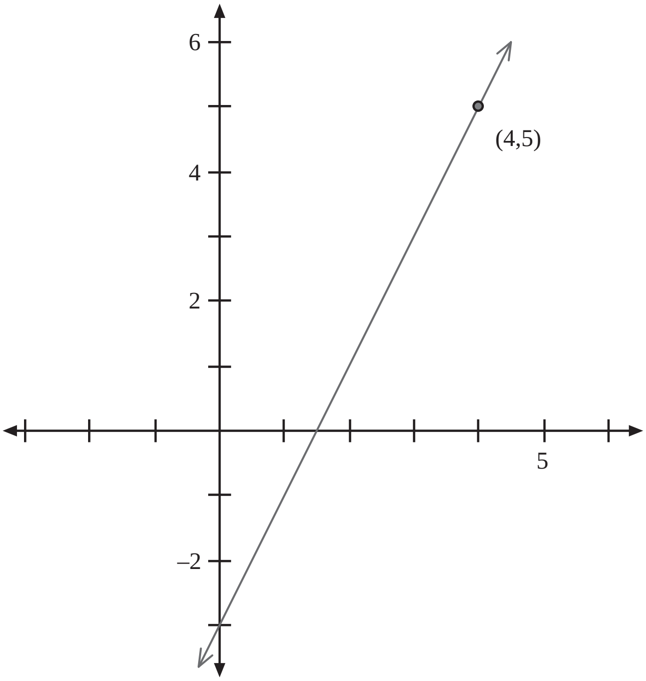
(d) If the slope is undefined, the line is vertical. All points will have the same x-coordinate. The equation is $x = -2$.

(e) First find the slope of the line $2x - 3y = 8$, because parallel lines have the same slope. Two points on this line are $(4, 0)$ and $\left(0, -\frac{8}{3}\right)$. So, the slope is $\frac{-\frac{8}{3}-0}{0-4} = \frac{-\frac{8}{3}}{-4} = \frac{8}{3} \cdot \frac{1}{4} = \frac{2}{3}$. The point (x, y) is on the line if

$$\begin{aligned} \frac{y-5}{x-(-2)} &= \frac{2}{3} \\ y - 5 &= \frac{2}{3}(x+2) \\ y - 5 &= \frac{2}{3}x + \frac{4}{3} \\ y &= \frac{2}{3}x + \frac{4}{3} + 5 \\ y &= \frac{2}{3}x + \frac{19}{3} \end{aligned}$$

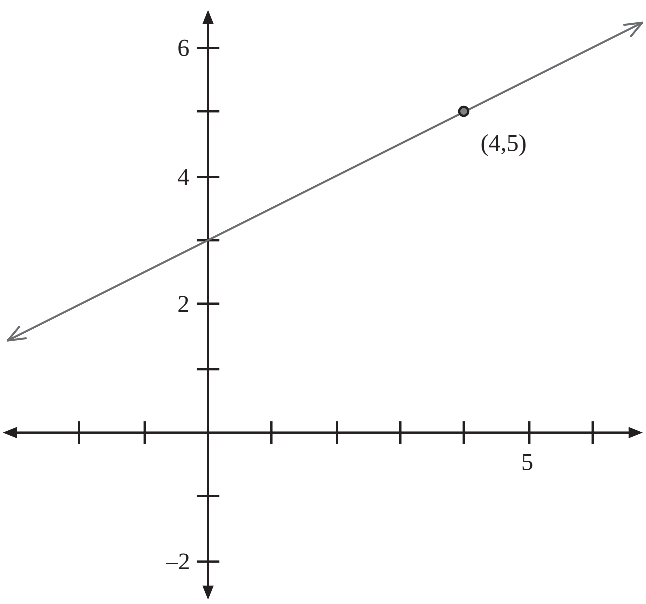
3. (a) $(4, 5)$ is on the line because

$$\begin{aligned} y &= 2x - 3 \\ 5 &\stackrel{?}{=} 2(4) - 3 \\ 5 &\stackrel{?}{=} 8 - 3 \\ 5 &= 5 \end{aligned}$$



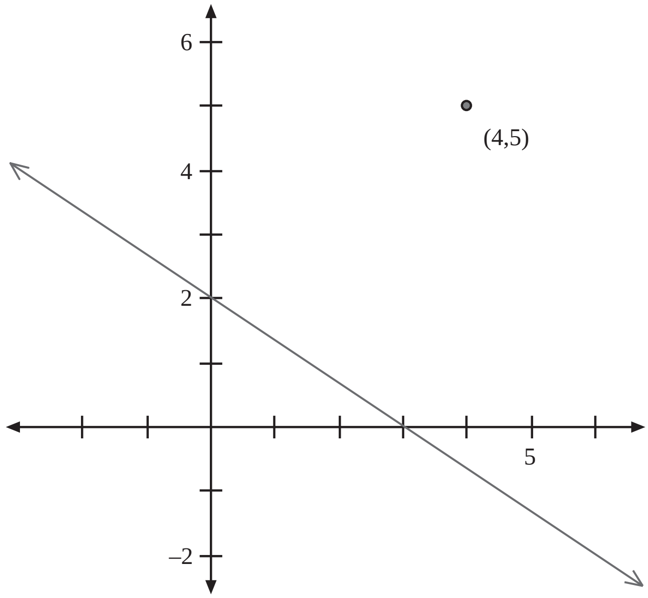
(b) $(4, 5)$ is on the line because

$$\begin{aligned} y - 1 &= \frac{1}{2}(x + 4) \\ 5 - 1 &\stackrel{?}{=} \frac{1}{2}(4 + 4) \\ 4 &\stackrel{?}{=} \frac{1}{2}(8) \\ 4 &= 4 \end{aligned}$$



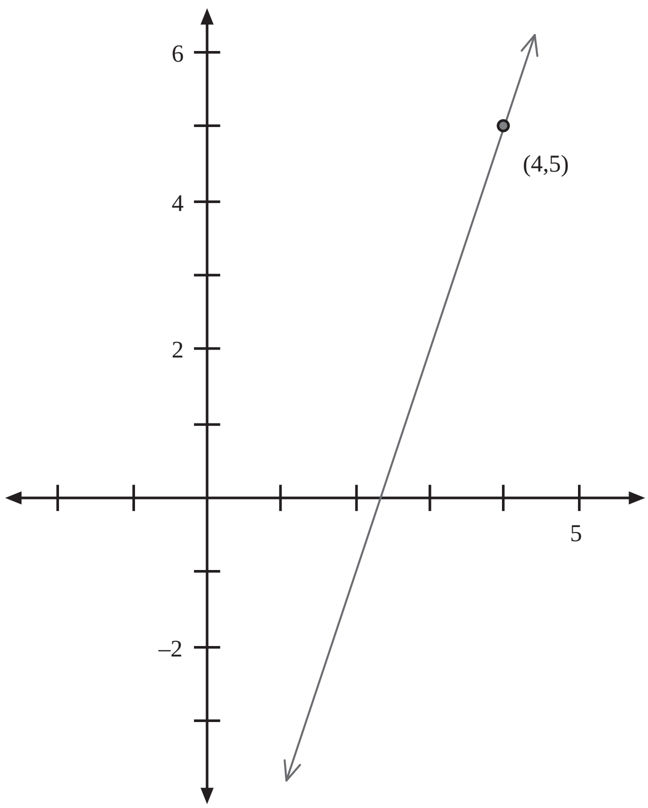
(c) $(4, 5)$ is not on the line because

$$\begin{aligned} 2x + 3y &= 6 \\ 2(4) + 3(5) &\stackrel{?}{=} 6 \\ 8 + 15 &\stackrel{?}{=} 6 \\ 23 &\neq 6 \end{aligned}$$



(d) $(4, 5)$ is on the line because

$$\begin{aligned} 3x - y &= 7 \\ 3(4) - 5 &\stackrel{?}{=} 7 \\ 12 - 5 &\stackrel{?}{=} 7 \\ 7 &= 7 \end{aligned}$$



4. (a)

$$\begin{aligned} 2x + y &= 5 \\ 2x - 2x + y &= 5 - 2x \\ y &= -2x + 5 \end{aligned}$$

(b)

$$\begin{aligned} 3x - y &= 4 \\ 3x - 3x - y &= 4 - 3x \\ -y &= -3x + 4 \\ y &= 3x - 4 \end{aligned}$$

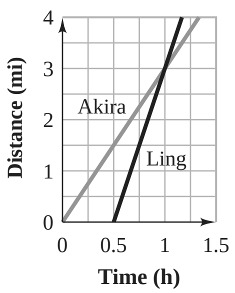
(c)

$$\begin{aligned} x + 2y &= -6 \\ x - x + 2y &= -6 - x \\ 2y &= -x - 6 \\ y &= -\frac{1}{2}x - 3 \end{aligned}$$

(d)

$$\begin{aligned} y - 4 &= 3(x - 2) \\ y - 4 &= 3x - 6 \\ y &= 3x - 6 + 4 \\ y &= 3x - 2 \end{aligned}$$

5. (a)



(b) The distance that Ling runs is her rate, 6 miles per hour, multiplied by the time, t, that she is running. Akira's distance is her rate, 3 miles per hour, multiplied by her time walking, which is $\frac{1}{2}$ hour more than Ling's time, or $t + \frac{1}{2}$. You want to know when the distances are equal.

$$\begin{aligned} 6t &= 3\left(t + \frac{1}{2}\right) \\ 6t &= 3t + \frac{3}{2} \\ 6t - 3t &= 3t - 3t + \frac{3}{2} \\ 3t &= \frac{3}{2} \\ \frac{1}{3} \cdot 3t &+ \frac{3}{2} \cdot \frac{1}{3} \\ t &= \frac{1}{2} \end{aligned}$$

6. No matter what form the equation of a line is, it can be changed using the basic rules and moves to an equation of the form $ax + by = c$, where a, b, and c are constants.

7. Graph each runner's distance versus time on the same graph. To find when one runner overtakes the other, look for the point of intersection.

8. First find the slope:

$$\frac{7-2}{-3-4} = \frac{5}{-7}$$

Use a point-tester equation with (x, y) and one of the given points:

$$\begin{aligned} \frac{y-2}{x-4} &= \frac{5}{-7} \\ y - 2 &= -\frac{5}{7}(x - 4) \\ 7(y - 2) &= 7 \cdot -\frac{5}{7}(x - 4) \\ 7y - 14 &= -5(x - 4) \\ 7y - 14 &= -5x + 20 \\ 7y - 14 + 5x &= -5x + 5x + 20 \\ 5x + 7y &= 34 \end{aligned}$$

MID-CHAPTER TEST

Multiple Choice

1. The correct answer is **D**, $2x + y = 10$.
A line with negative slope moves down as it goes to the right. Only line D moves down and to the right.
More specifically, line A has slope 2, line B has slope 0 (it is horizontal), line C has slope 1 (its equation is equivalent to $y = x + 5$), and line D has slope -2 (its equation is equivalent to $y = -2x + 10$).

2. The correct answer is **A**, $-\frac{4}{5}$.
Slope between two points is calculated as the change in y, divided by the change in x. Going from point A to point B, the change in y is $-4 = 1 - 5$, and the change in x is $5 = 2 - (-3)$. The slope is $-\frac{4}{5}$.
Graphing the two points shows that the slope should be negative: point A is above and to the left of point B.

3. The correct answer is **B**, $(y + 7) = 3(x - 10)$.
Using $x = 10$ and $y = -7$ as a point-tester works here. The only equation that is true when $x = 10$ and $y = -7$ is **B**.
This is also a translation of the line $y = 3x$. Moving it ten units right and seven down gives the equation $(y + 7) = 3(x - 10)$.

4. Find the slope between point A and each of the points in the answer choices:

A: $m = \frac{1-5}{10-4} = \frac{-4}{6} = \frac{-2}{3}$
B: $m = \frac{3-5}{7-4} = \frac{-2}{3}$
C: $m = \frac{7-5}{1-4} = \frac{2}{-3}$
D: $m = \frac{8-5}{2-4} = \frac{3}{-2}$

The slope between A and the point in answer choice D is not $\frac{-2}{3}$, so the coordinates in answer choice D cannot be the coordinates of point B. The correct answer is **D**.

5. The correct answer is **A**, between O and P.
Since distance is on the vertical, Dario travels the farthest when the change in distance is greatest. This happens between O and P. There is no change in distance between P and Q, and the changes in distance in the last two segments are smaller.
6. The correct answer is **C**, between Q and S.
Dario is traveling the fastest when the rate of change (Δ distance divided by Δ time) is the greatest. This is the slope. The slope between Q and S is the steepest, so it gives the fastest rate of change.
7. The correct answer is **D**, $y = -3x + 6$.
The graph goes through the points $(0, 6)$ and $(2, 0)$. One way to find the answer is to find the equation that is true for both these points: only **D** is true for both. Another way is to find the slope between the given points: it is -3. Knowing this, and that the graph goes through $(0, 6)$, means the equation is $y = -3x + 6$.

Open Response

8. (a) Answers will vary. You could use point-tester to prove that the line passes through $(2, -4)$. Some common solutions are $y = -2x$, $y = -4$, and $y = x - 6$.
(b) Answers will vary. If the given slope of the line is m, the value of y when $x = -20$ is

$$y = -22m - 4$$

9. No, Sean's reasoning is not correct.
Answers may vary. Sample: $A = (0, 0)$, $B = (1, 2)$, $C = (-1, -10)$ have the right slopes from A to B and from B to C, but the slope from A to C is 10, not 4.
10. There are two ways to find some points:
First, you could write an equation for the line. One is $(y - 2) = -\frac{1}{4}(x - 6)$, which can be simplified to $y = -\frac{1}{4}x + \frac{7}{2}$. Then find points by selecting any value for x and finding y. Some of those points include $(0, \frac{7}{2})$, $(2, 3)$,$(10, 1)$,$(14, 0)$, and others.
Alternatively, you could find points directly by repeating the slope. Going from R to S is down 2 units, and to the right 8 units. Continuing gives $(14, 0)$, then $(22, -2)$, then $(30, -4)$. Continuing in the opposite direction gives $(-10, 6)$, then $(-18, 8)$, then $(-26, 10)$. All these points are collinear, since the slope between them and R or S matches the slope between R and S.
11. (a) Caleb has traveled 150 miles in two hours. This is an average speed of 75 miles per hour, which is speeding.
(b) Caleb will drive 250 miles in total. If his average speed is 65 miles per hour, he will be on the road for $\frac{250 \text{ miles}}{65 \text{ mi/hr}} \approx 3.846$ hours. So far, Caleb has traveled for 2 hours. He has 100 miles left to go, and must take at least 1.846 hours to drive it. The average speed must be no more than

$$\frac{100 \text{ miles}}{1.846 \text{ hours}} \approx 54 \text{ mi/hr}$$

INVESTIGATION 4C INTERSECTIONS

4.09 Getting Started

For You to Explore

1. You can use the Guess-Check-Generalize method to solve the exercise. First, guess 10. After selling 10 T-shirts,
- Tuan will have $\$5.75(10) = \$57.50$
- He will have spent $\$1.25(10) + \$72.00 = \$84.50$

Tuan will not have broken even, because $\$57.50 \neq \$84.50$.
Follow the same procedure with a variable guess. Suppose Tuan has sold s T-shirts, then
- he will have $5.75(s)$ dollars
- he will have spent $1.25(s) + 72$ dollars

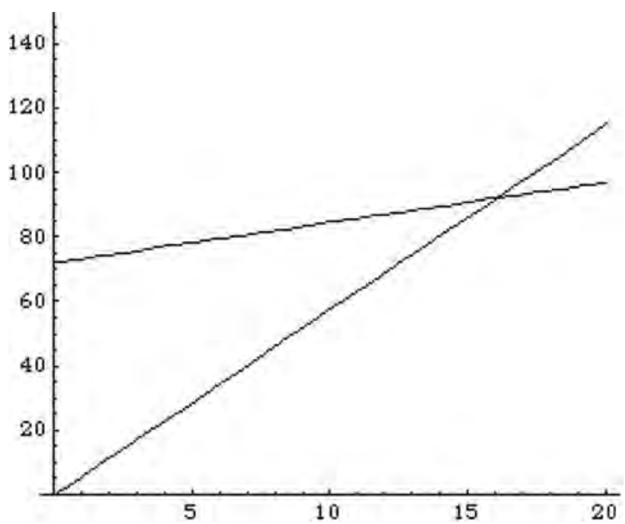
He will break even when the amount he makes equals the amount he has sold, i.e., when

$$5.75(s) = 1.25(s) + 72$$

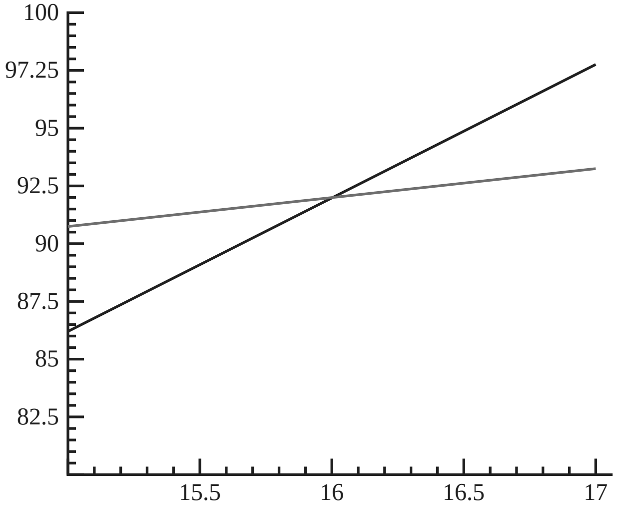
You can solve this equation using the basic moves. Subtract

$$\begin{aligned} 5.75(s) &= 1.25(s) + 72 \\ 4.5(s) &= 72 \\ s &= \frac{72}{4.5} = 16 \end{aligned}$$

So, Tuan will break even after selling 16 T-shirts.
You could also notice that Tuan makes $(5.75 - 1.25 = 4.50)$ profit for each T-shirt he sells. He will break even when the profits are equal to the initial amount he spent to set up the process, \$72. So the number of T-shirts he needs to sell to break even would be $\frac{72 \text{ dollars}}{4.5 \text{ dollars/shirt}} = 16$ T-shirts.
2. In this investigation, you will learn some more efficient methods for solving this type of problem. One way you could start is by inspecting the graphs:



The intersection seems to be near $x = 15$. Take a closer look:



Now that you can see the intersection better, it appears that the intersection point might be at $x = 16$. Test whether that works by putting $x = 16$ into the first equation:

$$\begin{aligned} y &= 72 + 1.25x \\ y &= 72 + 1.25(16) \\ y &= 72 + 22.5 \\ y &= 92 \end{aligned}$$

The point $(16, 92)$ does satisfy the first equation. Now test $x = 16$ in the second equation:

$$\begin{aligned} y &= 5.75x \\ y &= 5.75(16) \\ y &= 92 \end{aligned}$$

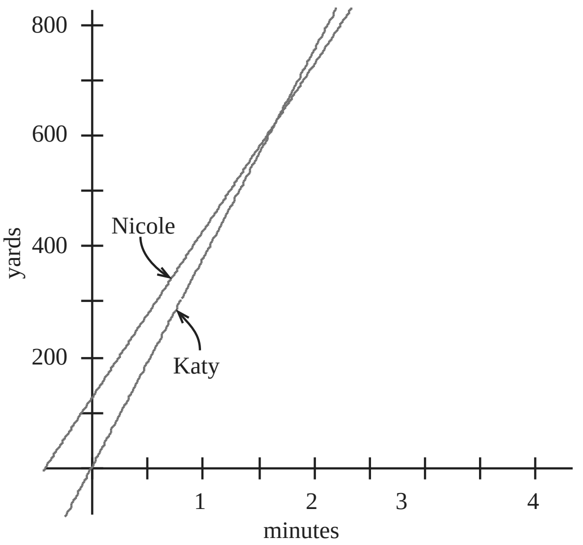
The point $(16, 92)$ also satisfies the second, so $(16, 92)$ is the intersection of the graphs of those two equations.

3. The solution for this exercise closely follows the solution to Exercise 1. Adjust the equation from that solution by replacing \$72.00 with \$130.00, and solve the new equation as you did before using the basic moves.

$$\begin{aligned} 5.75s &= 1.25s + 130 \\ 4.5s &= 130 \\ s &= \frac{130}{4.5} = \frac{260}{9} = 28\frac{8}{9} \end{aligned}$$

Tuan cannot sell $\frac{8}{9}$ of a T-shirt. Remember, Tuan makes \$4.50 in profit for each shirt he sells. If he sells 28 T-shirts, he makes $\$4.50(28) = \$126$, less than \$130. If he sells 29 T-shirts, he makes $\$4.5(29) = \$130.50$, more than \$130.

4. (a)



Nicole was given a head start of 125 yards, so at time $t = 0$, Nicole should be at location $d = 125$. This point, $(0, 125)$, is on the less steep graph.

Also, since Katy is moving faster than Nicole, her rate of change (with respect to location) is faster than Nicole's. As you saw earlier in the chapter, a greater rate of change signifies a greater slope, and the line with the greater slope has a steeper graph.

(b) Katy's position at time 0 is distance 0, so the point $(0, 0)$ is on the graph. She is traveling at a constant rate of 375 yd/min. So, using the point-tester method to write an equation of a line, it is a line that passes through $(0, 0)$ and has slope 375:

$$\begin{aligned} \frac{(d-0)}{(t-0)} &= 375 \\ \frac{d}{t} &= 375 \\ d &= 375t \end{aligned}$$

(c) As shown in the solution to part (a), the point $(0, 125)$ is on the graph of Nicole's equation, since at time 0 she is at distance 125 yards. She travels at a constant rate of 300 yd/sec. So, again using the point-tester method, it is a line that passes through $(0, 125)$ with slope 300:

$$\begin{aligned} \frac{(d-125)}{(t-0)} &= 300 \\ \frac{(d-125)}{t} &= 300 \\ (d-125) &= 300t \\ d &= 300t + 125 \end{aligned}$$

(d) There are several different ways to solve this equation.

- You could try plugging in different values of t until you found the time when Katy and Nicole are at the same location.
- You could reason that every minute Katy gains 75 yards on Nicole. So, she will have gained 125 yards (Nicole's head start) after $\frac{125}{75}$ minutes.
- You could use guess–check–generalize to find an equation to solve.
  First, make a guess: how about 10 minutes? Calculate Katy's location at $t = 10$.

$$\begin{aligned} d &= 375t \\ d &= 375(10) \\ d &= 3750 \end{aligned}$$

  Then calculate Nicole's location.

$$\begin{aligned} d &= 125 + 300t \\ d &= 125 + 300(10) \\ d &= 3125 \end{aligned}$$

  After 10 minutes, are Katy and Nicole at the same location? No! Because $3750 \neq 3125$.
  Try another guess, like $t = 5$. Katy's location is

$$\begin{aligned} d &= 375t \\ d &= 375(5) \\ d &= 1875 \end{aligned}$$

  and Nicole's location is

$$\begin{aligned} d &= 125 + 300t \\ d &= 125 + 300(5) \\ d &= 1625 \end{aligned}$$

  After 5 minutes, are Katy and Nicole at the same location? No, because $1875 \neq 1625$. Again, you didn't find the answer, but if you follow your steps, you have a method for checking any guess.
  Try t for your next guess. Calculate Katy's location: $375t$.
  Then calculate Nicole's location: $125 + 300t$.
  The next step is to see if they are equal. Well, you want them to be equal, so set the two expressions equal to each other:

$$375t = 125 + 300t$$

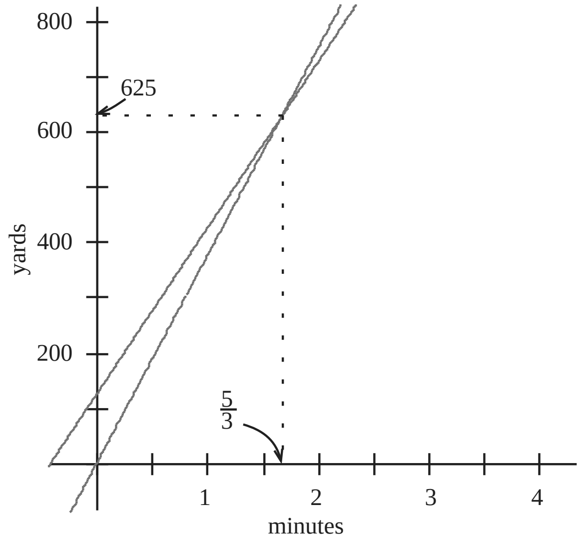
  Now solve this equation using the basic moves:

$$\begin{aligned} 375t &= 125 + 300t \\ 75t &= 125 \\ t &= \frac{125}{75} = \frac{5}{3} = 1\frac{2}{3} \end{aligned}$$

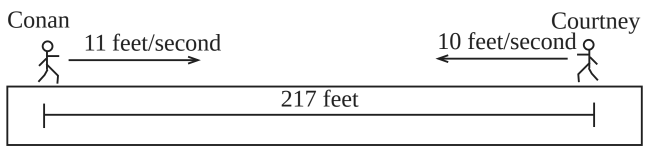
So, Katy will catch Nicole at time $t = \frac{5}{3}$.
To find their distance from the starting line, substitute this value of t back into one of the equations—you can use either one, but Katy's is easier:

$$\begin{aligned} d &= 375t \\ d &= 375\left(\frac{5}{3}\right) \\ d &= 625 \end{aligned}$$

So, Katy will catch Nicole after she has peddled 625 yards.



5. This picture shows Conan and Courtney on the train platform.



(a) Conan and Courtney are running toward each other. After 1 second, Conan has run 11 feet and Courtney has run 10 feet, so they are 21 feet closer to each other. After 2 seconds, they will be $21 \text{ ft/s} \cdot 2 \text{ s} =$ 42 feet closer to each other.

Suppose they meet after t seconds. Then they will have run $21t = 217$, since that is how far apart they were when they began. Solve this equation using the basic moves:

$$\begin{aligned} 21t &= 217 \\ t &= \frac{217}{21} = \frac{31}{3} = 10\frac{1}{3} \end{aligned}$$

(b) Conan runs 11 feet per second, so after t seconds, he has run $11t$ feet. He meets Courtney after $10\frac{1}{3}$ seconds, so Conan will have run

$$11 \text{ ft/s}\left(10\frac{1}{3} \text{ s}\right) = 113\frac{2}{3} \text{ feet.}$$

(c) Depending on how you answered parts (a) and (b), you may already have come up with an equation. The solution above suggests one equation, $21t = 217$, but there are many equations you could come up with.

Another way to think about the situation is to write two separate equations for Conan and Courtney. Suppose they run along a number line, where Conan starts at 0, and Courtney starts at 217. Let t be the number of seconds they run.

Conan has run $11t$ feet after t seconds. He starts at 0, so an expression for his location would be $11t + 0$, or simply $11t$.

Courtney runs $10t$ feet after t seconds. She starts at 217, and she is running toward zero, so an expression for her location would be $217 - 10t$.

They meet when the two expressions are equal, that is, when $11t = 217 - 10t$.

(d) Adjust your equation from part (c), replacing 217 with 350, to get a new equation equivalent to $11t = 350 - 10t$. Solve this equation using the basic moves:

$$\begin{aligned} 11t &= 350 - 10t \\ 21t &= 350 \\ t &= \frac{350}{21} = \frac{50}{3} = 16\frac{2}{3} \end{aligned}$$

On Your Own

6. Answers may vary, but a complete answer should include the following elements:

- How fast is each person running (or biking, or driving, etc.)?
- How much of a head start does one person get?
- The graphs should reflect the answers to these two questions.

7. Test each point by substituting it into each equation.

(a) $\ell : y = 2x + 1$, $m : y + 3x = 1$

$A = (0, 0)$
$\ell : (0) \neq 2(0) + 1$
$m : (0) + 3(0) \neq 1$
so A is on neither line.

$B = (0, 1)$
$\ell : (1) = 2(0) + 1$
$m : (1) + 3(0) = 1$
so B is on both lines.

$C = (1, \frac{1}{3})$
$\ell : (\frac{1}{3}) \neq 2(1) + 1$
$m : (\frac{1}{3}) + 3(1) \neq 1$
so C is on neither line.

(b) $\ell : y = x^2$, $m : \frac{1}{2}y + x = 4$

$A = (-2, 4)$
$\ell : (4) = (-2)^2$
$m : \frac{1}{2}(4) + (-2) \neq 4$
so A is on line ℓ, but not on line m.

$B = (2, 4)$
$\ell : (4) = (2)^2$
$m : \frac{1}{2}(4) + (2) = 4$
so B is on both lines.

$C = (0, 0)$
$\ell : (0) = (0)^2$
$m : \frac{1}{2}(0) + (0) \neq 4$
so C is on line ℓ, but not on line m.

(c) $\ell : y = \frac{1}{5}x - 1$, $m : x = 5$.

$A = (5, 0)$
$\ell : (0) = \frac{1}{5}(5) - 1$
$m : (5) = 5$
So A is on both lines.

$B = (5, 3)$
$\ell : (3) \neq \frac{1}{5}(5) - 1$
$m : (5) = 5$
So B is on line m, but not on line ℓ.

$C = (-5, 2)$
$\ell : (2) \neq \frac{1}{5}(-5) - 1$
$m : (-5) \neq 5$
So C is on neither line.

(d) $\ell : y = 2x$, $m : y = \frac{3}{2}x + 1$.

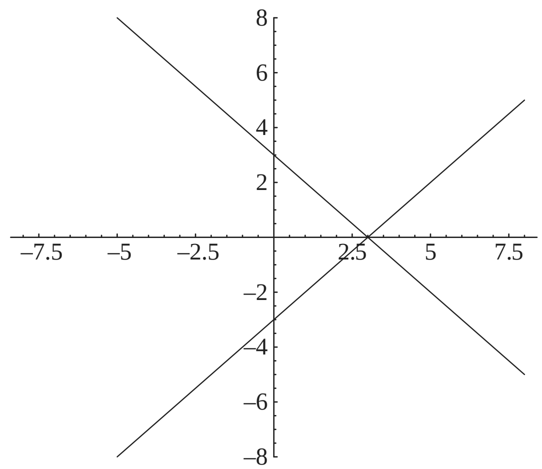
$A = (0, 1)$
$\ell : (1) \neq 2(0)$
$m : (1) = \frac{3}{2}(0) + 1$
A is on m only.

$B = (6, 10)$
$\ell : (10) \neq 2(6)$
$m : (10) = \frac{3}{2}(6) + 1$
B is on m only.

$C = (2, 4)$
$\ell : (4) = 2(2)$
$m : (4) = \frac{3}{2}(2) + 1$
C is on both.

8. Your answers to this exercise will vary, but the answers to part (iii) should always be the same. There are a number of different ways you could find answers to part (iii). This solution will illustrate how to solve by graphing.

(a) First, graph the two lines:



i. Finding a point on neither line is easiest, since you can choose a point that is far away from both lines. For example, it is pretty clear that $(0, 0)$ is on neither line. You can double-check by plugging the point into both equations and making sure it satisfies neither one:

$$\begin{aligned} y + 3 &= x \\ (0) + 3 &\stackrel{?}{=} (0) \\ 3 &\neq 0. \end{aligned}$$

So $(0, 0)$ is not on line p.

$$\begin{aligned} x + y &= 3 \\ (0) + (0) &\stackrel{?}{=} 3 \\ 0 &\neq 3. \end{aligned}$$

And $(0, 0)$ is also not on line q.

ii. To find a point that is on one line but not the other, pick a point that looks like it is on one line, but nowhere near the other one. For instance, at $x = 0$, it is clear that line p is below the x-axis, but line q is above the x-axis. The point on p at $x = 0$ is

$$\begin{aligned} y + 3 &= x \\ y + 3 &= (0) \\ y &= -3 \end{aligned}$$

So the point $(0, -3)$ is on line p. Make sure $(0. -3)$ is not on q:

$$\begin{aligned} x + y &= 3 \\ (0) + (-3) &\stackrel{?}{=} 3 \\ -3 &\neq 3 \end{aligned}$$

So $(0, -3)$ is not on q.

iii. Finding a point on both lines is a little trickier. From the figure, the two graphs seem to intersect at $(3, 0)$. If this point is the point of intersection, it must satisfy both equations:

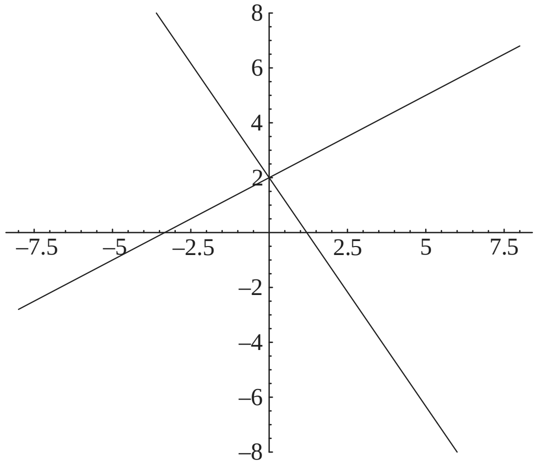
$$\begin{aligned} y + 3 &= x \\ (0) + 3 &\stackrel{?}{=} (3) \\ 3 &= 3 \end{aligned}$$

So $(3, 0)$ is on line p.

$$\begin{aligned} x + y &= 3 \\ (3) + (0) &\stackrel{?}{=} 3 \\ 3 &= 3 \end{aligned}$$

And $(3, 0)$ is also on q, so it must be the point of intersection of p and q.

(b) Graph the two lines, and have a look:



i. The point (0,0) looks like it is on neither line. And, indeed, the point satisfies neither equation.

$$\begin{aligned} y &= \frac{3}{5}x + 2 \\ (0) &\stackrel{?}{=} \frac{3}{5}(0) + 2 \\ 0 &\stackrel{?}{=} 0 + 2 \\ 0 &\neq 2 \end{aligned}$$

So $(0, 0)$ is not on line p.

$$\begin{aligned} y &= -\frac{5}{3}x + 2 \\ (0) &\stackrel{?}{=} -\frac{5}{3}(0) + 2 \\ 0 &\stackrel{?}{=} 0 + 2 \\ 0 &\neq 2 \end{aligned}$$

And $(0, 0)$ is also not on line q.

ii. The two lines are nowhere near each other around $x = 5$, so find the point on line p at $x = 5$:

$$\begin{aligned} y &= \frac{3}{5}x + 2 \\ y &= \frac{3}{5}(5) + 2 \\ y &= 3 + 2 \\ y &= 5 \end{aligned}$$

So the point $(5, 5)$ is on p, but not on q. Double-check that it is not on q by substituting it in and seeing that you get a false statement:

$$\begin{aligned} y &= -\frac{5}{3}x + 2 \\ (5) &\stackrel{?}{=} -\frac{5}{3}(5) + 2 \\ 5 &\stackrel{?}{=} -\frac{25}{3} + 2 \\ 5 &\neq -\frac{19}{3} \end{aligned}$$

iii. The graphs seem to intersect at $(0, 2)$. Plug this point into each equation to make sure it satisfies both equations.

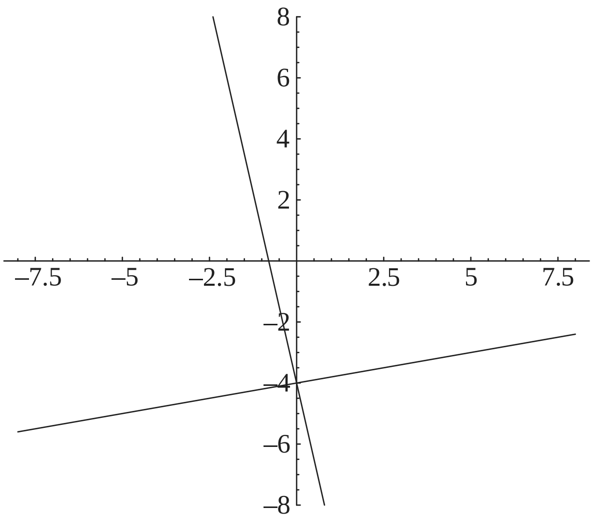
$$\begin{aligned} y &= \frac{3}{5}x + 2 \\ (2) &\stackrel{?}{=} \frac{3}{5}(0) + 2 \\ 2 &\stackrel{?}{=} 0 + 2 \\ 2 &= 2 \quad \checkmark \end{aligned}$$

So $(0, 2)$ is on line p.

$$\begin{aligned} y &= -\frac{5}{3}x + 2 \\ (2) &\stackrel{?}{=} -\frac{5}{3}(0) + 2 \\ 2 &\stackrel{?}{=} 0 + 2 \\ 2 &= 2 \quad \checkmark \end{aligned}$$

And $(0, 2)$ is also on line q.

(c) Graph the two equations:



i. Once more, $(0, 0)$ seems to be far away from both graphs. And, again, the point satisfies neither equation. (You should check the point with both equations yourself.)

ii. The graphs are far apart at $x = -1$. The point on p at $x = -1$ is $(-1, 1)$. (You should check that the point is on p but not on q.)

iii. The graphs intersect at $(0, -4)$. Plug this point into each equation to make sure that it satisfies both equations.

9. You can select any number of lines that intersect at $(2, -4)$. This solution will show how to answer the question if you choose the lines $x + y = -2$ and $2x + y = 0$.
Start with $x + y = -2$ and plug in $(2, -4)$:

$$\begin{aligned} x + y &= -2 \\ (2) + (-4) &\stackrel{?}{=} -2 \\ -2 &= -2 \end{aligned}$$

So, $(2, -4)$ satisfies the first equation. Now, plug this point into the second equation:

$$\begin{aligned} 2x + y &= 0 \\ 2(2) + (-4) &\stackrel{?}{=} 0 \\ 4 + (-4) &\stackrel{?}{=} 0 \\ 0 &= 0 \quad \checkmark \end{aligned}$$

So, $(2, -4)$ also satisfies the second equation.

10. First write two equations that represent the population of each town in the year 2002:

$$\begin{aligned} \text{Town X} &: 7250 - 120p \\ \text{Town Y} &: 9000 + 300p \end{aligned}$$

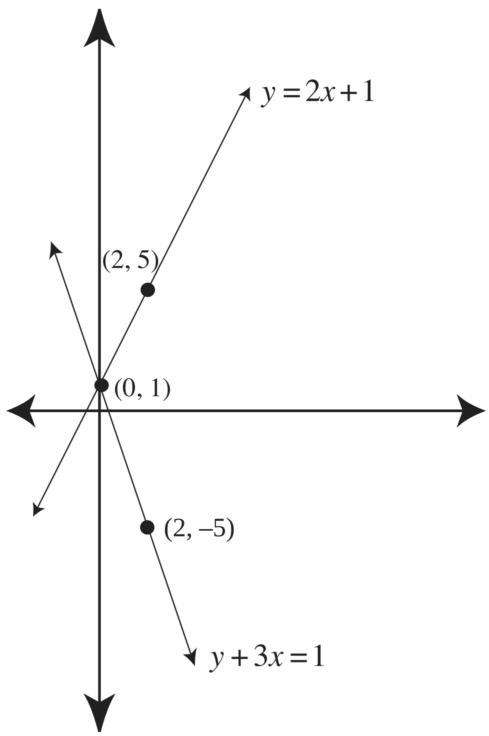
Now, set the two expressions equal to each other to find out when the populations were the same:

$$\begin{aligned} 7250 - 120p &= 9000 + 300p \\ -1750 &= 420p \\ p &= \frac{-1750}{420} \\ p &= -\frac{25}{6} = -4\frac{1}{6} \end{aligned}$$

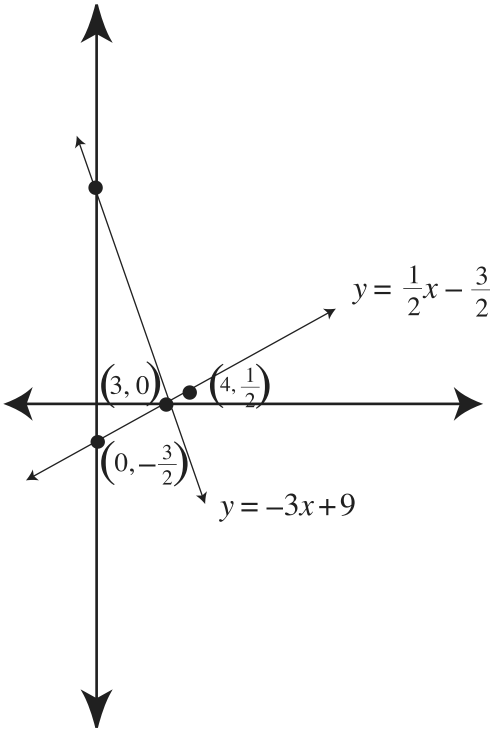
So, over 4 years before 2002 the populations were the same, sometime in the year 1997.

Maintain Your Skills

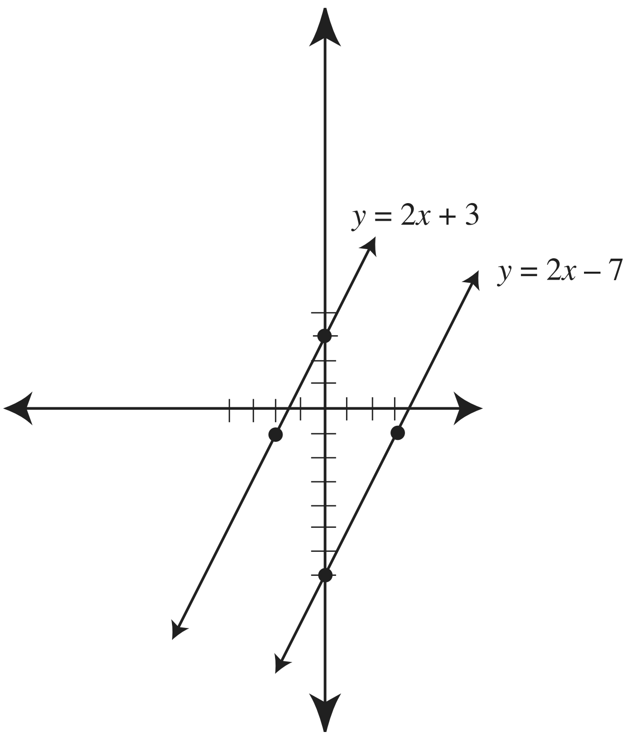
11. (a) These two lines intersect at the point $(0, 1)$. They have the point $(0, 1)$ in common.



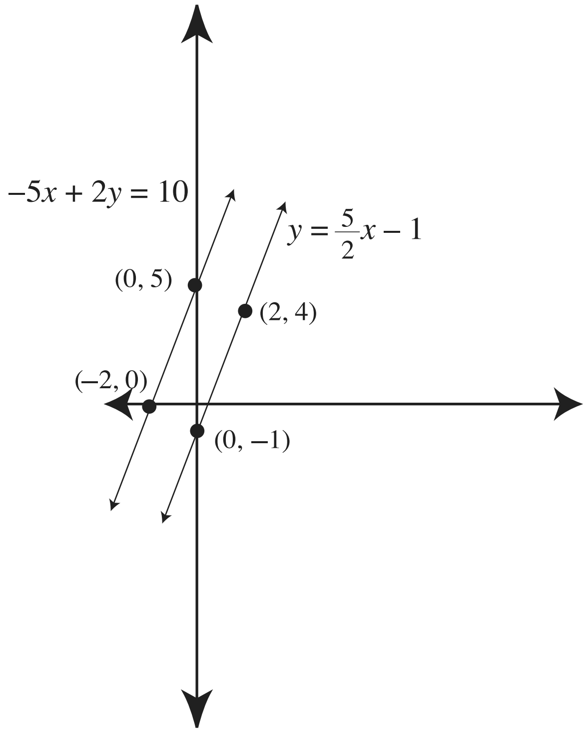
(b) These two lines intersect at the point $(3, 0)$. They have the point $(3, 0)$ in common.



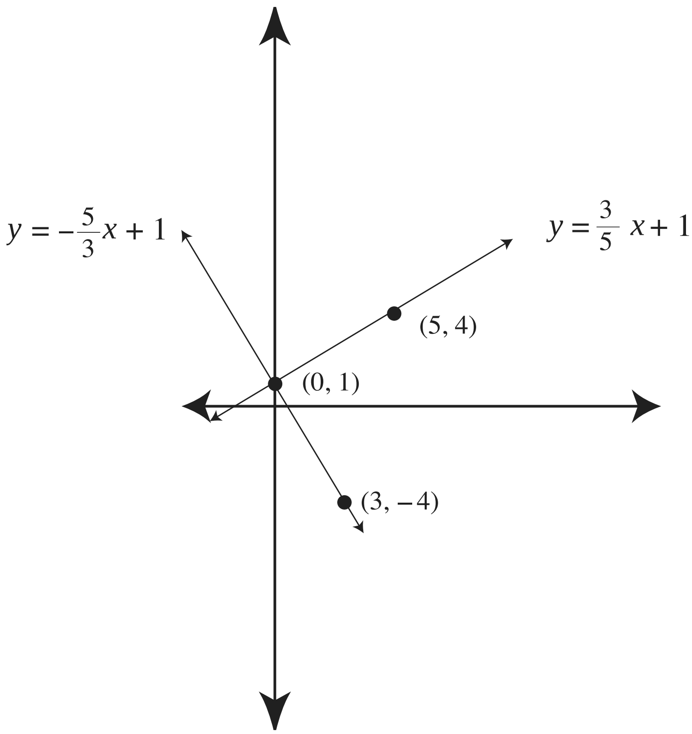
(c) These two lines do not intersect. They are *parallel* and have no points in common.



(d) These two lines do not intersect. They are *parallel* and have no points in common.



(e) These two lines intersect at the point $(0, 1)$—in fact, they seem to intersect at a 90° angle. They have the point $(0, 1)$ in common.



(f) Two linear equations have one intersection point when they have different slopes.

(g) Two linear equations have no intersection point when they have the same slope.

4.10 Solving Systems: Substitution

Check Your Understanding

1. First, use variables to write equations for the situation. Let v stand for veggie wraps and a for apples. Since you are told a drink costs \$.99, you don't need a variable for drinks—just use this number. Aisha's bill was \$6.24 for 2 veggie wraps, 3 apples, and a drink, so

$$2v + 3a + 0.99 = 6.24$$

Jaime's bill was \$6.64 for 3 veggies wraps, 2 apples, and a drink, so

$$3v + 2a + 0.99 = 6.64$$

Start by subtracting 0.99 from both sides of both equations:

$$\begin{aligned} 2v + 3a + 0.99 &= 6.24 \\ 2v + 3a &= 5.25 \end{aligned}$$

And

$$\begin{aligned} 3v + 2a + 0.99 &= 6.64 \\ 3v + 2a &= 5.65 \end{aligned}$$

Solve the first equation for a.

$$\begin{aligned} 2v + 3a &= 5.25 \\ 3a &= 5.25 - 2v \\ a &= 1.75 - \frac{2}{3}v \end{aligned}$$

Now, plug this expression into the second equation wherever you see a.

$$\begin{aligned} 3v + 2a &= 5.65 \\ 3v + 2\left(1.75 - \frac{2}{3}v\right) &= 5.65 \\ 3v + 3.5 - \frac{4}{3}v &= 5.65 \\ \frac{5}{3}v + 3.5 &= 5.65 \\ \frac{5}{3}v &= 2.15 \\ v &= 2.15\left(\frac{3}{5}\right) = 1.29 \end{aligned}$$

So, veggie wraps cost \$1.29. How much do apples cost? You can plug 1.29 for v into either equation, but you already solved for a above, so use that equation.

$$\begin{aligned} a &= 1.75 - \frac{2}{3}(1.29) \\ a &= 1.75 - 0.86 \\ a &= \frac{2.67}{3} = 0.89 \end{aligned}$$

So an apple costs \$.89, and a veggie wrap costs \$1.29.

2. This exercise uses the same equations as Exercise 1, so the solution is the same: $v = 1.29$, and $a = 0.89$.

3. (a) With BigPhone, a six-minute call would cost \$.39 for the connection plus \$.03(6) for six minutes. The total is

$$0.39 + 0.03(6) = 0.39 + 0.18 = 0.57$$

With LittlePhone, a six-minute call would cost \$.25 for the connection plus \$.07(6) for six minutes. The total is:

$$0.25 + 0.07(6) = 0.25 + 0.42 = 0.67$$

So, on BigPhone a six-minute call costs \$.57, but on LittlePhone a six-minute call costs \$.67.

(b) Let m be the number of minutes you spent on the call. If c is the cost of the phone call, then BigPhone's plan can be written as the equation

$$c = 0.39 + 0.03m$$

and LittlePhone's plan could be written as the equation:

$$c = 0.25 + 0.07m$$

The plans will cost the same when the graphs of these two equations intersect. Since both equations are written in $c =$ form, you can set their right sides equal, and solve that equation to find the intersection point:

$$\begin{aligned} 0.39 + 0.03c &= 0.25 + 0.07c \\ 0.14 + 0.03c &= 0.07c \\ 0.14 &= 0.04c \\ \frac{0.14}{0.04} &= c \\ 3.5 &= c \end{aligned}$$

So, a $3\frac{1}{2}$ minute call would cost the same on both plans.

(c) With BigPhone, a ten-minute call would cost \$.39 for the connection plus \$.03(10) for ten minutes. The total is

$$0.39 + 0.03(10) = 0.39 + 0.30 = 0.69$$

With LittlePhone, a ten-minute call would cost \$.25 for the connection plus \$.07(10) for ten minutes. The total is

$$0.25 + 0.07(10) = 0.25 + 0.70 = 0.95$$

So, on BigPhone a ten-minute call costs \$.69, but on LittlePhone a ten-minute call costs \$.95.

4. (a) Since both equations are solved for y, set the right sides equal to each other.

$$\begin{aligned} 6x - 7 &= -2x - 9 && \text{Add } 2x \text{ to each side.} \\ 8x - 7 &= -9 && \text{Add 7 to each side.} \\ 8x &= -2 && \text{Divide each side by 8.} \\ x &= -\frac{1}{4} \end{aligned}$$

Plug your answer back into either equation to get y.

$$y = -2x - 9$$
$$y = -2\left(-\frac{1}{4}\right) - 9$$
$$y = \frac{1}{2} - 9$$
$$y = -8\frac{1}{2}$$

Check your answer of $\left(-\frac{1}{4}, -8\frac{1}{2}\right)$ by plugging the values into the original equations.

$$-8\frac{1}{2} = 6 \cdot \left(-\frac{1}{4}\right) - 7 \quad \checkmark$$
$$-8\frac{1}{2} = -2 \cdot \left(-\frac{1}{4}\right) - 9 \quad \checkmark$$

(b) Again, both equations are solved for m, so set the right sides equal to each other.

$$8n - 3 = -4n + 6 \qquad \text{Add } 4n \text{ to each side.}$$
$$12n - 3 = 6 \qquad \text{Add 3 to each side.}$$
$$12n = 9 \qquad \text{Divide each side by 12.}$$
$$n = \frac{3}{4}$$

Plug the answer into either equation to get m.

$$m = 8n - 3$$
$$m = 8\left(\frac{3}{4}\right) - 3$$
$$m = 6 - 3$$
$$m = 3$$

Check your answer of $\left(\frac{3}{4}, 3\right)$ by plugging the values into the original equations.

$$3 = 8 \cdot \left(\frac{3}{4}\right) - 3 \quad \checkmark$$
$$3 = -4 \cdot \left(\frac{3}{4}\right) + 6 \quad \checkmark$$

(c) The first equation tells you that y and $3x$ are equal, so plug in $3x$ every time you see y in the second equation.

$$3x - 2(3x) = 12 \qquad \text{Multiply } -2 \cdot 3.$$
$$3x - 6x = 12 \qquad \text{Add } 3x \text{ and } -6x.$$
$$-3x = 12 \qquad \text{Divide each side by } -3.$$
$$x = -4$$

Then plug in that value for x in the first equation to find y.

$$y = 3x$$
$$y = 3(-4)$$
$$y = -12$$

Check your answer of $(-4, -12)$ by plugging the values into the original equations.

$$3 = 12 = 3 \cdot -4 \quad \checkmark$$
$$3 = 3 \cdot -4 - 2 \cdot -12 = 12 \quad \checkmark$$

(d) Solve both equations for y, then set them equal to each other. The first equation becomes

$$(y - 5) = 3(x + 2) \qquad \text{Distribute the 3.}$$
$$y - 5 = 3x + 6 \qquad \text{Add 5 to each side.}$$
$$y = 3x + 11$$

The second equation becomes

$$(y - 5) = 6(x + 2) \qquad \text{Distribute the 6.}$$
$$y - 5 = 6x + 12 \qquad \text{Add 5 to each side.}$$
$$y = 6x + 17$$

Now, set the left sides equal to each other to get the equation:

$$3x + 11 = 6x + 17 \qquad \text{Subtract } 3x \text{ from each side.}$$
$$11 = 3x + 17 \qquad \text{Subtract 17 from each side.}$$
$$-6 = 3x \qquad \text{Divide each side by 3.}$$
$$-2 = x$$

Plug this into either equation to get y (use the first, after you've solved for y):

$$y = 3x + 11$$
$$y = 3(-2) + 11$$
$$y = -6 + 11$$
$$y = 5$$

Test your answer of $(-2, 5)$ by plugging it into the original equations.

$$(5 - 5) = 3(-2 + 2) \quad \checkmark$$
$$(5 - 5) = 6(-2 + 2) \quad \checkmark$$

(e) Both equations are solved for y, so set the right sides equal to each other.

$$9x + 4 = 9x - \frac{2}{3} \qquad \text{Subtract } 9x \text{ from both sides.}$$
$$4 = -\frac{2}{3} \qquad \text{Oops! That's not right.}$$

The last statement is false, so there are no common solutions.

(f) First, solve one of the equations for y:

$$(y - 1) = \frac{2}{3}(x + 1) \qquad \text{Distribute the } \tfrac{2}{3}.$$
$$y - 1 = \frac{2}{3}x + \frac{2}{3} \qquad \text{Add 1 to each side.}$$
$$y = \frac{2}{3}x + \frac{5}{3}$$

Then subsitute this expression into the second equation:

$$\begin{aligned}
(y-1) &= 4\left(x-\frac{1}{4}\right) \\
\left(\left(\frac{2}{3}x+\frac{5}{3}\right)-1\right) &= 4\left(x-\frac{1}{4}\right) && \text{Add } \tfrac{5}{3} \text{ and } -1. \\
\frac{2}{3}x+\frac{2}{3} &= 4\left(x-\frac{1}{4}\right) && \text{Distribute the 4.} \\
\frac{2}{3}x+\frac{2}{3} &= 4x-1 && \text{Add 1 to each side.} \\
\frac{2}{3}x+\frac{5}{3} &= 4x && \text{Subtract } \tfrac{2}{3}x \text{ from each side.} \\
\frac{5}{3} &= \frac{10}{3}x && \text{Multiply each side by } \tfrac{3}{10}. \\
\frac{1}{2} &= x
\end{aligned}$$

Now, plug this value for x into either equation (using the one you solved for y might be easiest):

$$\begin{aligned}
y &= \frac{2}{3}x+\frac{5}{3} \\
y &= \frac{2}{3}\left(\frac{1}{2}\right)+\frac{5}{3} \\
y &= \frac{1}{3}+\frac{5}{3} \\
y &= \frac{6}{3} \\
y &= 2
\end{aligned}$$

Check your answer of $\left(\frac{1}{2}, 2\right)$ by plugging the point into both equations.

$$\begin{aligned}
(y-1) &= \frac{2}{3}(x+1) \\
(2-1) &\stackrel{?}{=} \frac{2}{3}\left(\frac{1}{2}+1\right) \\
1 &\stackrel{?}{=} \frac{2}{3}\cdot\frac{1}{2}+\frac{2}{3} \\
1 &\stackrel{?}{=} \frac{1}{3}+\frac{2}{3} \\
1 &= 1 \quad \checkmark
\end{aligned}$$

$$\begin{aligned}
(y-1) &= 4\left(x-\frac{1}{4}\right) \\
(2-1) &\stackrel{?}{=} 4\left(\frac{1}{2}-\frac{1}{4}\right) \\
1 &\stackrel{?}{=} \frac{4}{1}\cdot\frac{1}{2}-\frac{4}{1}\cdot\frac{1}{4} \\
1 &\stackrel{?}{=} 2-1 \\
1 &= 1 \quad \checkmark
\end{aligned}$$

5. (a) A $\frac{3}{10}$-mile ride using Walton would be $2.25 + 0.19 \times 3 = 2.82$.
A $\frac{3}{10}$-mile ride using Newtham would be $1.50 + 0.25 \times 3 = 2.25$.

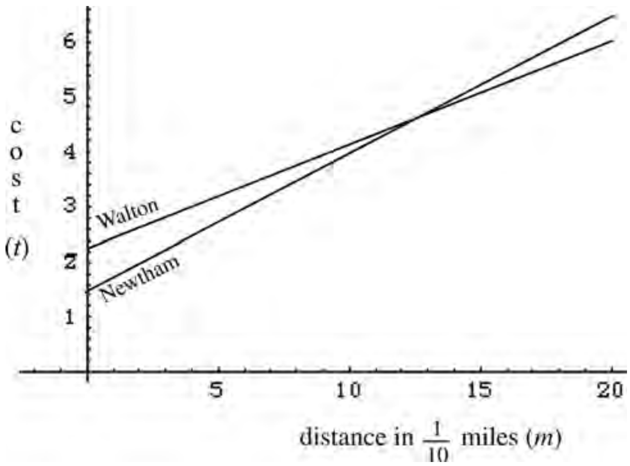
(b) One mile is $\frac{10}{10}$ so,
A one-mile ride using Walton would be

$$2.25 + 0.19 \times 10 = 4.15$$

A one-mile ride using Newtham would be

$$1.50 + 0.25 \times 10 = 4.00$$

(c) Walton: $t = 2.25 + 0.19m$
Newtham: $t = 1.5 + 0.25m$



(d) The graphs of these two lines have the point $(12.5, 3.125)$ in common. So each taxi company would charge \$3.13 for $12\frac{1}{2}$ one-tenth mile segments—or $1\frac{1}{4}$ miles.

(e) Six miles is 60 tenth-mile segments. According to the graph, the Walton company will be cheaper for this distance.

(f) For any distance over $1\frac{1}{4}$ mile, it will be cheaper to use the Walton company.

6. To find the intersection point, you first need to find an equation for each line.

(a) The slope of line k is $m(k) = \frac{6-0}{-4-2} = \frac{6}{-6} = -1$.
The line goes through the point $(2, 0)$, so write a point-tester equation: $y - 0 = -1(x - 2)$. You can simplify the equation to

$$y = -x + 2$$

The slope of line ℓ is $m(\ell) = \frac{6-3}{7-2} = \frac{3}{5}$.
The line goes through the point $(2, 3)$, so write a point-tester equation:

$$y - 3 = \frac{3}{5}(x - 2)$$

You solved line k's equation for y, so you can replace y with $-x + 2$ in line ℓ's equation to find the intersection point.

$$
\begin{aligned}
(-x+2)-3 &= \frac{3}{5}(x-2) && \text{Simplify the left side.}\\
-x-1 &= \frac{3}{5}(x-2) && \text{Multiply each side by 5.}\\
-5x-5 &= 3(x-2) && \text{Distribute the 3.}\\
-5x-5 &= 3x-6 && \text{Subtract } 3x \text{ from each side.}\\
-8x-5 &= -6 && \text{Add 5 to each side.}\\
-8x &= -1 && \text{Divide each side by } -8.\\
x &= \frac{1}{8}
\end{aligned}
$$

Now, plug $\frac{1}{8}$ in for x in the first equation.

$$
\begin{aligned}
y &= -x+2\\
y &= -\left(\frac{1}{8}\right)+2\\
y &= \frac{15}{8}
\end{aligned}
$$

So the intersection point is $\left(\frac{1}{8}, \frac{15}{8}\right)$. Test it out by plugging it into the original equations.

$$
k: \quad \frac{15}{8} = -\frac{1}{8}+2 \quad \checkmark
$$

$$
\begin{aligned}
\ell: \quad \frac{15}{8} &\stackrel{?}{=} \frac{3}{5}\cdot\frac{1}{8}+\frac{9}{5}\\
\frac{15}{8} &\stackrel{?}{=} \frac{3}{40}+\frac{72}{40}\\
\frac{15}{8} &\stackrel{?}{=} \frac{75}{40}\\
\frac{15}{8} &= \frac{15}{8} \quad \checkmark
\end{aligned}
$$

(b) n is a horizontal line so the equation for n is $y = 3$.

The slope of line o is $m(o) = \frac{5\frac{1}{2}-(-2)}{7-0} = \frac{\frac{15}{2}}{7} = \frac{15}{14}$. o crosses the y-axis at -2 so the point $(0, -2)$ is on the line. The point-tester equation for the line is then

$$
y+2 = -\frac{15}{14}x
$$

The equation for n gives you a value for y, so plug 3 into the equation for o to get the value for x.

$$
\begin{aligned}
(3)+2 &= \frac{15}{14}x && \text{Simplify the left side.}\\
5 &= \frac{15}{14}x && \text{Multiply each side by } \tfrac{14}{15}.\\
5\cdot\frac{14}{15} &= x\\
\frac{14}{3} &= x
\end{aligned}
$$

So the intersection point is $\left(\frac{14}{3}, 3\right)$. Test it out by plugging the point into the two original equations. The point works for line n, because $3 = 3$.

$$
\begin{aligned}
o: \quad 3+2 &\stackrel{?}{=} \frac{15}{14}\left(\frac{14}{3}\right)\\
5 &\stackrel{?}{=} \frac{15}{3}\\
5 &= 5 \quad \checkmark
\end{aligned}
$$

(c) The slope of line p is $m(p) = \frac{10-1}{0-10} = -\frac{9}{10}$. The graph shows that the line crosses the y-axis at 10, so the point $(0, 10)$ is on the graph. The point-tester equation is then

$$
y-10 = -\frac{9}{10}x
$$

The slope of line q is $m(q) = \frac{0-(-2)}{0-(-2)} = 1$. The graph shows that q goes through the origin, so the point $(0, 0)$ is on the graph. The point-tester is then simple to write:

$$
y = x
$$

The equation for q is solved for y, so you can just plug x in for y in the equation for p and solve.

$$
\begin{aligned}
y-10 &= -\frac{9}{10}x && \text{Replace } y \text{ with } x.\\
(x)-10 &= -\frac{9}{10}x && \text{Add 10 to each side.}\\
x &= -\frac{9}{10}x+10 && \text{Add } \tfrac{9}{10} \text{ to each side.}\\
\frac{19}{10}x &= 10 && \text{Multiply each side by } \tfrac{10}{19}.\\
x &= \frac{100}{19}
\end{aligned}
$$

So the intersection point is $\left(\frac{100}{19}, \frac{100}{19}\right)$. Test it by plugging it into the two original equations. (Well, it is easy to see that it works for line q, since the two coordinates are the same!)

$$
\begin{aligned}
p: y-10 &= -\frac{9}{10}x\\
\left(\frac{100}{19}\right)-10 &\stackrel{?}{=} -\frac{9}{10}\left(\frac{100}{19}\right)\\
\frac{100}{19}-\frac{190}{19} &\stackrel{?}{=} -\frac{90}{19}\\
-\frac{90}{19} &= -\frac{90}{19} \quad \checkmark
\end{aligned}
$$

7. There are many different answers to this question. Exercise 5 part (d), Exercise 3 part (b), and Exercise 1 are all good examples.

8. Answers will vary.

On Your Own

9. The jobs are priced by the half hour, so convert the length of the job to half-hour units.

(a) 1 hour is two half hours.

RotoPlumb $75 for the first half hour plus $30 for the other half hour makes the total job cost $\$75 + \$30 = \$105$.

JustPlumbing $45 for each half hour times 2 half-hours makes the job cost $\$45 \cdot 2 = \$90$.

So, it will be cheaper for Tyrell to hire JustPlumbing for this short job.

(b) 6 hours is 12 half hours.

RotoPlumb $75 for the first half hour, plus $30 for another 11 half hours is
$\$75 + 11 \cdot \$30 = \$75 + \$330 = \$405.$

JustPlumbing $45 for 12 half hours is
$\$45 \cdot 12 = \$540.$

So, it will be cheaper for Tyrell to hire RotoPlumb for this longer job.

(c) First write an equation to represent each company's charges. Let c be the cost of the job, and h be the length of the job (in half hours):

RotoPlumb $c = 75 + 30(h - 1)$
Just Plumbing $c = 45h$

Both equations are solved for c, so you can set the right sides equal to each other to see where the graphs of these two equations intersect:

$$\begin{aligned} 75 + 30(h-1) &= 45h \\ 75 + 30h - 30 &= 45h \\ 45 + 30h &= 45h \\ 45 &= 15h \\ h &= 3 \end{aligned}$$

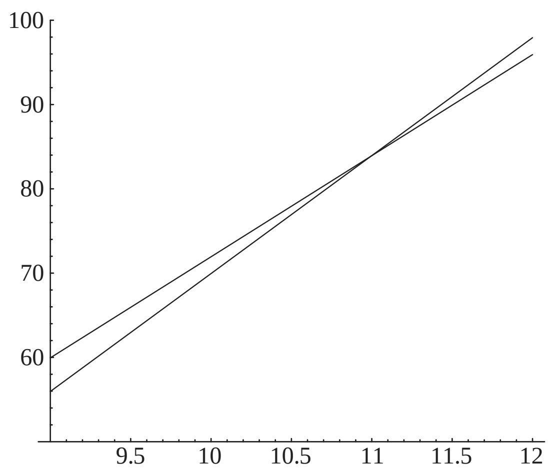
The two companies will charge the same amount for a 3 half hours (or $1\frac{1}{2}$ hour) job, at a cost of $135.

10. (a) For Club #1, you get 4 DVDs free, so you pay for 5 DVDs. The cost is then $\$11.99 \cdot 5 = \$59.95$.
For Club #2, you get 5 DVDs free, so you pay for 4 DVDs. The total cost is then $\$13.99 \cdot 4 = \$55.96$.

(b) One way to complete the table would be to add a row for 9 DVDs, then just add the cost of one more DVD to the total in each sucessive row.

Total Number of DVDs	Total cost from Club #1	Total Cost from Club #2
9	$59.95	$55.96
10	$71.94	$69.95
11	$83.93	$83.94
12	$95.92	$97.93

(c) The figure below shows the graphs for Club #1 and Club #2. The vertical axis is the cost of the club, and the horizontal axis is the number of DVDs purchased. The steeper line represents Club #2.



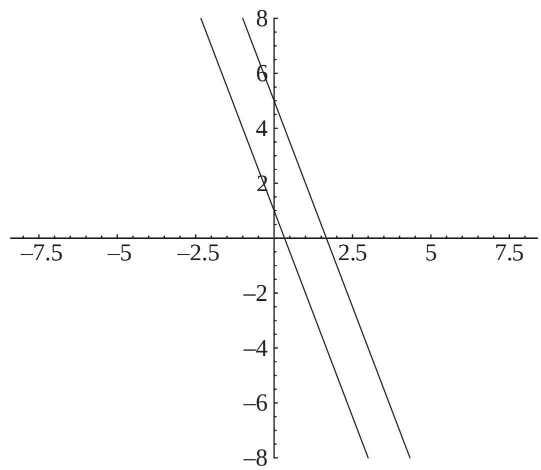
(d) Club #1 is more economical for large DVD purchases: specifically, it is cheaper when 11 or more DVDs are purchased during the year. Club #2 is more economical when purchasing either 9 or 10 DVDs, and only in those situations.

Notice that the horizontal axis begins at 9. Since the minimum purchase requirement for Club #2 is 9 DVDs, but for Club #1 it is 8 DVDs, Club #1 will be more economical, because with Club #2 you'll still have to pay for that ninth DVD. It does not really make sense to compare the clubs for purchases that are less than 8 DVDs, since the total cost will not go down.

11. (a) You end up with the equation $5 = 1$. If you subsitute $y = 5 - 3x$ into the second equation, you will get the following equation:

$$\begin{aligned} y + 3x &= 1 \\ (5 - 3x) + 3x &= 1 \\ 5 &= 1 \quad \text{Something must be wrong.} \end{aligned}$$

(b) There is no intersection point. It appears from the graph, though, that the lines are parallel. Since parallel lines do not intersect, the system of equations will not have a solution.



12. To find the intersection point, set the right sides of both equations equal:

$$18x + K = 5x + L$$

Now, solve for x in terms of K and L:

$$\begin{aligned} 18x + K &= 5x + L \\ 13x + K &= L \\ 13x &= L - K \\ x &= \frac{L - K}{13} \end{aligned}$$

So the x-coordinate of the intersection point is $\frac{L-K}{13}$. To find the y-coordinate, plug this value into either equation (the second one looks easier).

$$y = 5x + L$$
$$y = 5\left(\frac{L-K}{13}\right) + L$$
$$y = \frac{5L-5K}{13} + \frac{13L}{13}$$
$$y = \frac{18L-5K}{13}$$

So the intersection point is given by the formula $(x, y) = \left(\frac{L-K}{13}, \frac{18L-5K}{13}\right)$.

13. (a) Both equations are solved for y, so set the right sides equal to each other:

$$18x - 30 = 17.5x + 12$$
$$0.5x - 30 = 12$$
$$0.5x = 42$$
$$x = 84$$

Plug $x = 84$ into the second equation to find the y-value of the intersection point:

$$y = 18x + 12$$
$$y = 18(84) + 12$$
$$y = 1524$$

So the intersection point is $(84, 1524)$.

(b) This system of equations has no solution. The lines appear to be parallel. Watch what happens if you try to solve this system of equations:

$$18x - 30 = 18x + 12$$
$$-30 \stackrel{?}{=} 12$$

You get the "equation" $-30 = 12$, which is false, so the system of equations has no solutions.

Maintain Your Skills

14. (a) First, calculate the slope.
slope of $\ell = \frac{5-1}{-1-(-3)} = \frac{4}{2} = 2$
slope of $m = \frac{4-(-2)}{5-2} = \frac{6}{3} = 2$
So an equation for ℓ is $y - 1 = 2(x+3)$. Solve for y and simplify to get $y = 2x + 7$.
An equation for m is $y + 2 = 2(x-2)$. Plug $2x + 7$ in for y in the equation for m.

$$y + 2 = 2(x-2)$$
$$(2x+7) + 2 = 2(x-2)$$
$$2x + 9 = 2x - 4$$

Uh oh. If you subtract $2x$ from both sides, you end up with $9 = -4$, which is false. So the lines don't intersect, and thus must be parallel.

(b) Again, calculate the slope.
slope of $\ell = \frac{4-1}{10-(-5)} = \frac{3}{15} = \frac{1}{5}$
slope of $m = \frac{0-2}{5-15} = \frac{-2}{-10} = \frac{1}{5}$
So an equation for ℓ is $y - 1 = \frac{1}{5}(x+5)$.
An equation for m is $y = \frac{1}{5}(x-5)$. Plug $\frac{1}{5}(x-5)$ in for y in the equation for ℓ.

$$y - 1 = \frac{1}{5}(x+5)$$
$$\left(\frac{1}{5}(x-5)\right) - 1 = \frac{1}{5}(x+5)$$
$$\frac{1}{5}x - 1 - 1 = \frac{1}{5}x + 1$$
$$\frac{1}{5}x - 2 = \frac{1}{5}x + 1$$

Oops. Again, if you subtract $\frac{1}{5}x$ from both sides, you end up with the false statement $-2 = 1$. So these two lines do not intersect, and are thus parallel.

(c) slope of $\ell = \frac{\frac{1}{2}-(-4)}{0-3} = \frac{\frac{9}{2}}{-3} = \frac{9}{2} \cdot \left(-\frac{1}{3}\right) = -\frac{3}{2}$
slope of $m = 2$
An equation for ℓ is $y - \frac{1}{2} = \frac{9}{2}x$. Solve for y to get $y = \frac{9}{2}x + \frac{1}{2}$.
An equation for m is $y - 5 = 2x$. Solve for y to get $y = 2x - 5$.
Set the right sides of the two equations solved for y equal to each other and solve for x.

$$\frac{9}{2}x + \frac{1}{2} = 2x - 5 \quad \text{Multiply each side by 2.}$$
$$9x + 1 = 4x - 10 \quad \text{Subtract } 4x \text{ from both sides.}$$
$$5x + 1 = -10 \quad \text{Subtract 1 from both sides.}$$
$$5x = -11 \quad \text{Divide both sides by 5.}$$
$$x = -\frac{11}{5}$$

The question did not ask for the intersection point, just whether or not the lines were parallel. Since you got a solution for x, these two lines must intersect.

(d) slope of $\ell = \frac{7-(-1)}{7-(-1)} = \frac{8}{8} = 1$
slope of $m = \frac{-7-0}{0-7} = \frac{-7}{-7} = 1$
An equation for ℓ is $y + 1 = x + 1$, which you can simplify to $y = x$.
An equation for m is $y = x + 7$.
If you set the right sides equal to each other, you get the equation $x + 1 = x + 7$. But that equation has no solution, since if you subtract x from both sides, you get $1 = 7$, a false statement. So the two lines do not intersect, thus they must be parallel.

(e) slope of $\ell = \frac{11-7}{9-3} = \frac{4}{6} = \frac{2}{3}$
slope of $m = \frac{1-(-4)}{3-(-6)} = \frac{5}{9}$
An equation for ℓ is $y - 7 = \frac{2}{3}(x-3)$. Solve for y and simplify to get $y = \frac{2}{3}x + 5$.
An equation for m is $y - 1 = \frac{5}{9}(x-3)$. Solve for y and simplify to get $y = \frac{5}{9}x - \frac{2}{3}$.
Set the right sides of the simplified equations equal to each other to get

$$\frac{2}{3}x + 5 = \frac{5}{9}x - \frac{2}{3}$$

You do not have to actually solve the equation. By looking at it, you can see that the coefficient of x is different on both sides, so it will not disappear—you can subtract $\frac{5}{9}x$ from both sides, and end up with x on one side but not the other, which means that you can find some value for x. And, since you can solve for x, the two lines must intersect.

(f) slope of $\ell = \frac{12-5}{-3-4} = \frac{7}{-7} = -1$

slope of $m = \frac{\frac{3}{2}-\frac{1}{2}}{3-4} = \frac{1}{-1} = -1$

An equation for ℓ is $y - 5 = -1(x - 4)$. Solve for y and simplify to get $y = -x + 9$.

An equation for m is $y - 3 = -1(x - \frac{3}{2})$. Solve for y and simplify to get $y = -x + \frac{9}{2}$.

To find the intersection point, you'd start by solving the equation

$$-x + 9 = -x + \frac{9}{2}$$

Right away, notice that both sides have $-x$. If you add x to both sides, you end up with $9 = \frac{9}{2}$, which is a false statement, so there is no intersection point and thus the lines are parallel.

(g) slope of $\ell = \frac{-1-(-9)}{2-0} = \frac{8}{2} = 4$

slope of $m = \frac{4-0}{4-0} = \frac{4}{4} = 1$

An equation for ℓ is $y + 9 = 4x$, or $y = 4x - 9$.

An equation for m is $y = x$.

If you can solve the equation $x = 4x - 9$, the lines cannot be parallel, because you will have found the x-coordinate of the point of intersection. Since $x = 3$ works in the equation, the lines cannot be parallel.

(h) When the two lines had the same slope, you could not solve the system of equations, which means that the lines were parallel. In the next lesson, you will prove that if two lines have the same slope, they must be parallel.

4.11 Slope and Parallel Lines

Check Your Understanding

1. (a) The slope of both lines is 4, so these lines are parallel.

(b) The slope of the first line is $\frac{5}{3}$, the slope of the second line is $-\frac{3}{5}$, so these lines will intersect.

(c) The slope of the first line is $\frac{4}{12}$ or $\frac{1}{3}$, the slope of the second line is also $\frac{1}{3}$, so these lines are parallel.

(d) The slope of both lines is $-\frac{2}{3}$, so these lines are parallel.

(e) The slope of the first line is $\frac{1}{4}$ and the slope of the second line is -4, so these lines will intersect.

(f) The slope of the first line is -2 and the slope of the second line is -1, so these lines will intersect.

2. Tony is right. Since Huge Phone is more expensive at first, but cheaper per minute, eventually it will be the better deal.

Write down the prices for each plan as equations. Let m be the number of minutes you talk, and let c be the cost of the telephone call. BigPhone's plan can be represented by the equation

$$c = 0.39 + 0.03m$$

Huge Phone's plan can be represented by the equation

$$c = 10.00 + 0.01m$$

These lines are not parallel (BigPhone has slope 0.03 and Huge Phone has slope 0.01). And since Huge Phone is cheaper per minute, sooner or later that savings must add up enough so that Huge Phone costs the same as BigPhone. We do not need to find this intersection point to know that Tony is right. It follows from the fact that lines with different slopes will eventually intersect.
But, if we wanted to double check, we could find the intersection point:

$$\begin{aligned} 0.39 + 0.03m &= 10.00 + 0.01m \\ 0.03m &= 9.61 + 0.01m \\ 0.02m &= 9.61 \\ m &= 480.5 \end{aligned}$$

So, Huge Phone will give you a better deal after you have talked for 480.5 minutes.

3. Check students' work.

4. She doesn't get another line because multiplying both sides of an equation by 4 is one of the basic moves. Remember that, by definition, the basic moves of equations are moves that *do not change the solutions of an equation.* The line $2x + 3y = 7$ and the line $8x + 12y = 28$ have the same solutions, so the graphs of these two equations are the same.

On Your Own

5. Since the two lines are parallel, they have the same slope. The slope of the line with equation $y = -\frac{1}{5}x + 4$ is $-\frac{1}{5}$. So the problem boils down to finding the equation of a line that passes through the point $(10, 15)$ with slope $-\frac{1}{5}$. Use a point-tester equation.

$$y - 15 = -\frac{1}{5}(x - 10)$$

6. First, find the slope of the line with equation $2x - 4y = 7$. Remember that the slope of an equation of the form $ax + by = c$ is $-\frac{a}{b}$. So the slope of this line is $-\frac{2}{-4} = \frac{1}{2}$. So again, the problem boils down to finding the equation of a line that passes through the point $(-4, 3)$ with slope $\frac{1}{2}$. Use a point-tester equation.

$$y - 3 = \frac{1}{2}(x + 4)$$

7. First, find the slope of the line with equation $y - 5 = \frac{22}{7}(x - 8)$, which is $\frac{22}{7}$. Once again, the problem boils down to finding the equation of a line that passes through the point $\left(13, -\frac{22}{7}\right)$ with slope $\frac{22}{7}$. Use a point-tester equation.

$$y - \frac{22}{7} = \frac{22}{7}(x - 13)$$

You may notice that this equation looks an awful lot like the original equation. They are both in the same form and they both have the same slope, they just go through different points.

8. The slope of the line $y = ax + b$ is a. So a point-tester equation through the origin, point $(0, 0)$, would be $y - 0 = a(x - 0)$, which simplifies to $y = ax$.

9. (a) The graph of $y = 7$ has a slope of 0—it is a horizontal line. You could go through the effort of finding the slope, using one of the equation forms, plugging in the slope, and so forth. Or, you could realize that all horizontal lines are parallel to each other, and each one will have an equation in the form of $y = b$. The horizontal line that goes through the point $(5, -3)$ has the equation $y = -3$.

(b) Using the old methods on vertical lines is even more difficult—they have no slope! But, just like horizontal lines, all vertical lines are parallel to each other, and each one will have an equation in the form of $x = a$. The vertical line that goes through the point $(5, -3)$ has the equation $x = 5$.

10. Izzie is correct. The situation Meredith described could not happen. Let c be the cost of each CD and b be the cost of each book. Then the equations for the two days are

$$39b + 21c = 396 \quad \text{and} \quad 52b + 28c = 518$$

If you graph the two equations you will see that they represent parallel lines. Why does that matter? Well, try to solve the system of equations to determine the cost of one book and one CD. If you solve for c, the first equation becomes

$$c = -\frac{39}{21}b + \frac{396}{21}$$

and the second becomes

$$c = -\frac{52}{28}b + \frac{518}{28}$$

Set them equal to each other, and solve for b.

$$\begin{aligned} -\frac{39}{21}b + \frac{396}{21} &= -\frac{52}{28}b + \frac{518}{28} \\ -\frac{39}{21}b + \frac{52}{28}b &= \frac{518}{28} - \frac{396}{21} \end{aligned}$$

You have a bunch of big fractions there! The problem is that $\frac{39}{21}$ reduces to $\frac{13}{7}$, and $\frac{52}{28}$ *also* reduces to $\frac{13}{7}$. So the left side of the equation is equal to 0. But on the right side, $\frac{518}{28}$ reduces to $\frac{37}{14}$, and $\frac{396}{21}$ reduces to $\frac{132}{7}$, so the right side is *not* equal to 0. Since the system cannot be solved, the price of either the book or the CD (or both) must have changed between Monday and Tuesday, or Meredith got some other numbers wrong.

11. If a system of equations has no solution, this means that the graphs of these equations have no point of intersection. So, they must be parallel lines.

12. The exercise states that x is the cost of one can of chili, and y the cost of one jar of salsa. Julia bought 3 cans of chili and 2 jars of salsa, so her cost would be $3x + 2y$. Marcia's 2 cans of chili and 4 jars of salsa would cost $2x + 4y$. The only choice with these two expressions is **D**. But before you make the choice, finish the equations. Julia's cost is represented by $3x + 2y$, and you are given that the total cost is \$10.07, so the complete equation would be $3x + 2y = 10.07$. By the same process, Marcia's complete equation would be $2x + 4y = 12.98$. And **D** has those two equations.

13. (a) Solve the equation $6x = 12$ to get $x = 2$. Now, plug $x = 2$ into either equation:

$$\begin{aligned} 2x + 3y &= 7 \\ 2(2) + 3y &= 7 \\ 4 + 3y &= 7 \\ 3y &= 3 \\ y &= 1 \end{aligned}$$

So Diego finds the solution $(2, 1)$.

(b) Adding the same thing to both sides of an equation is a basic move. For instance, you can add 5 to both sides of an equation. The move Diego made is an extension of that. Rather than adding 5 to both sides of the first equation, he added 5 to the right side, and something that equals $5(4x - 3y)$ to the left side. It is a valid move, because he kept the first equation balanced.

Maintain Your Skills

14. (a) Plug $(0, -5)$ into the equation and solve for k.

$$\begin{aligned} y &= 2x + k \\ -5 &= 2(0) + k \\ -5 &= k \end{aligned}$$

So $k = -5$.

(b) Again, plug $(0, -5)$ into the equation and solve for k.

$$\begin{aligned} y &= 3x + k \\ (-5) &= 3(0) + k \\ -5 &= k \end{aligned}$$

Again, $k = -5$.

(c) Plug $(0, -5)$ into the equation and solve for k.

$$\begin{aligned} y &= 4x + k \\ (-5) &= 4(0) + k \\ -5 &= k \end{aligned}$$

One more time, $k = -5$.

(d) Finally, plug $(0, -5)$ into the equation and solve for k.

$$\begin{aligned} y &= 99x + k \\ (-5) &= 99(0) + k \\ -5 &= k \end{aligned}$$

So each time, including this one, $k = -5$.

(e) Other exercises before this one have hinted at the fact that, for equations written in the form $y = ax + b$, the a is the slope of the line, and the b is where the line crosses the y-axis. Since the point $(0, -5)$ is on the y-axis, in each case, the value for k will be the y-coordinate of the y-intercept, in this case -5.

15. (a) Plug $(-5, 0)$ into the equation and solve for k.

$$\begin{aligned} y &= 2x + k \\ (0) &= 2(-5) + k \\ 0 &= -10 + k \\ k &= 10 \end{aligned}$$

(b) Again, plug $(-5, 0)$ into the equation and solve for k.

$$\begin{aligned} y &= 3x + k \\ 0 &= 3(-5) + k \\ 0 &= -15 + k \\ k &= 15 \end{aligned}$$

(c) Plug $(-5, 0)$ into the equation and solve for k.

$$\begin{aligned} y &= 4x + k \\ 0 &= 4(-5) + k \\ 0 &= -20 + k \\ k &= 20 \end{aligned}$$

(d) Plug $(-5, 0)$ into the equation and solve for k.

$$\begin{aligned} y &= 99x + k \\ 0 &= 99(-5) + k \\ 0 &= -495 + k \\ k &= 495 \end{aligned}$$

(e) The pattern is not quite as clear as the one from Exercise 14. But there clearly is a pattern.

The point $(-5, 0)$ is on the x-axis, since $y = 0$. So this point is where the line will cross the x-axis. It turns out that the value for k is always the opposite of the product of the slope and the x-intercept. That is, if the line $y = ax + k$ goes through the point $(c, 0)$, then $k = -ac$. You can also find this result by plugging the point $(c, 0)$ into the basic equation, and solving for k.

$$\begin{aligned} y &= ax + k && \\ (0) &= a(c) + k && \text{Subtract } k \text{ from each side.} \\ -k &= ac && \text{Divide each side by } -1. \\ k &= -ac && \end{aligned}$$

4.12 Solving Systems: Elimination

Check Your Understanding

1. (a) Solve by elimination.

$$\begin{array}{rrcl} & x + y &=& 30 \\ (+) & x - y &=& 6 \\ \hline & 2x &=& 36 \\ & x &=& 18 \end{array}$$

$$\begin{aligned} x + y &= 30 \\ (18) + y &= 30 \\ y &= 12 \end{aligned}$$

$$\begin{aligned} x + y &= 30 \\ (18) + (12) &\stackrel{?}{=} 30 \\ 30 &= 30 \quad \checkmark \end{aligned}$$

$$\begin{aligned} x - y &= 6 \\ (18) - (12) &\stackrel{?}{=} 6 \\ 6 &= 6 \quad \checkmark \end{aligned}$$

(b) Solve by elimination

$$\begin{array}{rrcl} & -10a + 6b &=& 25 \\ (+) & 10a + 5b &=& 30 \\ \hline & 11b &=& 55 \\ & b &=& 5 \end{array}$$

$$\begin{aligned} 10a + 5b &= 30 \\ 10a + 5(5) &= 30 \\ 10a + 25 &= 30 \\ 10a &= 5 \\ a &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} -10a + 6b &= 25 \\ -10\left(\frac{1}{2}\right) + 6(5) &\stackrel{?}{=} 25 \\ -5 + 30 &= 25 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 10a + 5b &= 30 \\ 10\left(\frac{1}{2}\right) + 5(5) &\stackrel{?}{=} 30 \\ 5 + 25 &= 30 \quad \checkmark \end{aligned}$$

(c) Solve by elimination.

$$\begin{array}{rrcl} & 2x + y &=& 4 \\ (+) & x - y &=& 2 \\ \hline & 3x &=& 6 \\ & x &=& 2 \end{array}$$

$$\begin{aligned} x - y &= 2 \\ (2) - y &= 2 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} 2x + y &= 4 \\ 2(2) + (0) &\stackrel{?}{=} 4 \\ 4 + 0 &= 4 \quad \checkmark \end{aligned}$$

$$\begin{aligned} x - y &= 2 \\ (2) - (0) &= 2 \quad \checkmark \end{aligned}$$

(d) Solve by elimination.

$$\begin{array}{rrcl} & 2x - 3y &=& 17 \\ (+) & x + 3y &=& 1 \\ \hline & 3x &=& 18 \\ & x &=& 6 \end{array}$$

$$\begin{aligned} x + 3y &= 1 \\ (6) + 3y &= 1 \\ 3y &= -5 \\ y &- \frac{5}{3} \end{aligned}$$

$$
\begin{aligned}
2x - 3y &= 17\\
2(6) - 3\left(-\frac{5}{3}\right) &\stackrel{?}{=} 17\\
12 + 5 &= 17 \quad \checkmark
\end{aligned}
$$

$$
\begin{aligned}
x + 3y &= 1\\
(6) + 3\left(-\frac{5}{3}\right) &\stackrel{?}{=} 1\\
6 - 5 &= 1 \quad \checkmark
\end{aligned}
$$

(e) Solve by elimination.

$$
\begin{array}{rrl}
 & 4z - 5w &= 15\\
(-) & 4z + 2w &= -6\\
\hline
 & -7w &= 21\\
 & w &= -3
\end{array}
$$

$$
\begin{aligned}
4z + 2w &= 6\\
4z + 2(-3) &= -6\\
4z &= 0\\
z &= 0
\end{aligned}
$$

$$
\begin{aligned}
4z - 5w &= 15\\
4(0) - 5(-3) &\stackrel{?}{=} 15\\
0 + 15 &= 15 \quad \checkmark
\end{aligned}
$$

$$
\begin{aligned}
4z + 2w &= -6\\
4(0) + 2(-3) &\stackrel{?}{=} -6\\
0 - 6 &= -6 \quad \checkmark
\end{aligned}
$$

(f) You don't have to use the *elimination* method here—the *substitution* method might be more straightforward for you.

$$
\begin{array}{rrl}
 & y &= 7x + 1\\
(-) & y &= 15\\
\hline
 & 0 &= 7x - 14\\
 & 14 &= 7x\\
 & 2 &= x
\end{array}
$$

$$
\begin{aligned}
y &= 7x + 1\\
15 &\stackrel{?}{=} 7(2) + 1\\
15 &= 14 + 1 \quad \checkmark
\end{aligned}
$$

$$
\begin{aligned}
y &= 15\\
(15) &= 15 \quad \checkmark
\end{aligned}
$$

2. First, write two equations for the two combinations described in the problem. Let g be the cost of a granola bar, and d be the cost of a drink. The equations would then be $2g + 2d = 3.50$ and $2g + 4d = 6.00$. Solve the system by substitution.

$$
\begin{array}{rrl}
 & 2g + 2d &= 3.50\\
(-) & 2g + 4d &= 6.00\\
\hline
 & -2d &= -2.50\\
 & d &= 1.25
\end{array}
$$

So a drink costs \$1.25. Now, plug in 1.25 for d in either equation to find g, the cost of a granola bar.

$$
\begin{aligned}
2g + 2d &= 3.50\\
2g + 2(1.25) &= 3.50\\
2g + 2.50 &= 3.50\\
2g &= 1.00\\
g &= 0.50
\end{aligned}
$$

So the cost of a granola bar is \$.50.

3. (a) Isabel first rearranged the second equation so the like terms would line up when she combined the two equations.

(b) Then she subtracted the second equation from the first equation.

(c) She solved for y.

(d) Then she used the value of y to solve for x in the second equation.

(e) Finally, she checked to make sure the point she found satisfied *both* equations.

4. First, find the intersection of the two graphs. From that point, you can find the equations for the horizontal and vertical lines that pass through it.

To use elimination, you need the coefficients of one of the variables to be the same. You cannot immediately eliminate either variable with the equations as they are now. But if you multiply the first equation by 2 and the second by 5, the coefficient of y will then be the same, 10.

$$
\begin{array}{rrrl}
 & 4x & + 10y &= 34\\
(+) & 15x & - 10y &= 80\\
\hline
 & 19x & &= 114\\
 & x & &= 6
\end{array}
$$

Now, plug 6 in for x in either equation to get y.

$$
\begin{aligned}
2x + 5y &= 17\\
2(6) + 5y &= 17\\
12 + 5y &= 17\\
5y &= 5\\
y &= 1
\end{aligned}
$$

So the intersection point of the two graphs is $(6, 1)$.

(a) The equation of the horizontal line through the point $(6, 1)$ is $y = 1$.

(b) The equation of the vertical line through $(6, 1)$ is $x = 6$.

(c) The line $5x + 3y = 33$ also passes through the intersection point $(6, 1)$.

Remember that when you use the basic moves on an equation, you do not change the solution set of the equations. The same result applies when you use Theorem 4.5. Even though each equation represents an infinite number of solutions, since they are lines, when you add them together, the one point they have in common—their intersection point—will also satisfy the new equation (and it will, in fact, be the only point that will do so).

5. To solve a system of three equations and three unknowns, you need to apply elimination or substitution more than once. Remember that the elimination process is called that because you eliminate a variable, yielding an equation of one unknown that you can solve with the basic rules and moves. So when there are three unknowns, you want to eliminate two unknowns, one at a time.

The tricky part is keeping track of the equations you use for elimination. Otherwise, you could go around in circles, never eliminating that second variable.

So, for step 1, take the first two equations and eliminate one of the variables, say z. You need to multiply the second equation by 5 so the coefficients are the same.

$$\begin{array}{rrcl} & 2x + 3y + 5z & = & 11 \\ (-) & 5x - 5y + 5z & = & 5 \\ \hline & -3x + 8y \quad\quad & = & 6 \end{array}$$

So now you have a linear equation of two variables. Progress! Next, pick another combination of two equations, and *eliminate the same variable,* z. You might tend toward taking the second and third, but notice that the coefficient of z is the same in the first and third equation, so no multiplication is needed.

$$\begin{array}{rrcl} & 2x + 3y + 5z & = & 11 \\ (+) & 3x - 4y - 5z & = & 16 \\ \hline & 5x - \ \ y \quad\quad & = & 27 \end{array}$$

So now you have two equations of two unknowns. Apply elimination again on these two equations to eliminate one more variable, say y. To do so, multiply the second equation, $5x - y = 27$ by 8.

$$\begin{array}{rrcl} & -3x + 8y & = & 6 \\ (+) & 40x - 8y & = & 216 \\ \hline & 37x \quad\quad & = & 222 \\ & x \quad\quad & = & 6 \end{array}$$

Next, plug 6 in for x in either of these two equations, say, $-3x + 8y = 6$.

$$\begin{aligned} -3x + 8y &= 6 \\ -3(6) + 8y &= 6 \\ -18 + 8y &= 6 \\ 8y &= 24 \\ y &= 3 \end{aligned}$$

Finally, plug 6 in for x and 3 in for y in any of the three original equations to get z.

$$\begin{aligned} x - y + z &= 1 \\ (6) - (3) + z &= 1 \\ 3 + z &= 1 \\ z &= -2 \end{aligned}$$

So the solution is $x = 6$, $y = 3$, and $z = -2$.

6. The process you have followed so far was to choose one of the variables to eliminate, then multiply the two equations by whatever numbers you need to make the coefficients the same. If you choose to eliminate x, you'll want to multiply the first equation by c and the second by a and then subtract. So the first equation is

$$\begin{aligned} c \cdot (ax + by) &= c \cdot e \\ c \cdot ax + c \cdot by &= c \cdot e \\ acx + bcy &= ce \end{aligned}$$

The second equation likewise becomes $acx + ady = af$.

Now, subtract the two equations to eliminate the x variable.

$$\begin{array}{rrcl} & acx + \quad\quad ady & = & af \\ (-) & acx + \quad\quad bcy & = & ce \\ \hline & ady - bcy & = & af - ce \end{array}$$

Now, you need to solve for y. You can undistribute the y on the left side, since $ady - bcy = (ad - bc)y$.

$$\begin{aligned} (ad - bc)y &= af - ce \\ y &= \frac{af - ce}{ad - bc} \end{aligned}$$

You have two choices now for finding the expression for x. You can plug this value for y into either equation and solve for x, or you can repeat the process you just followed, but this time eliminate the y. The easier way to go is to start back at the beginning.

Multiply the first equation by d and the second by b, then subtract. So the first equation is

$$\begin{aligned} d \cdot (ax + by) &= d \cdot e \\ d \cdot ax + d \cdot by &= d \cdot e \\ adx + bdy &= de \end{aligned}$$

The second equation likewise becomes $bcx + bdy = bf$.

Now, subtract the two equations to eliminate the y variable.

$$\begin{array}{rrcl} & adx + bdy & = & de \\ (-) & bcx + bdy & = & bf \\ \hline & adx - bcx \quad\quad & = & de - bf \end{array}$$

Now, you need to solve for x. Like you did before, undistribute the x on the left side, since $adx - bcx = (ad - bc)x$.

$$\begin{aligned} (ad - bc)x &= de - bf \\ x &= \frac{de - bf}{ad - bc} \end{aligned}$$

So the intersection point is

$$(x, y) = \left(\frac{de - bf}{ad - bc}, \frac{af - ce}{ad - bc}\right)$$

Note that if $ad = bc$, then this expression for the intersection point has a problem, since the denominator of both the x- and y-coordinates is 0. But if you go back to the original process, you'll see that when you try to solve the system, all the x and y terms vanish, giving the equation

$$0 = ce - af$$

In this case, if $ce = af$, then the equations are the same line. If not, they are parallel lines, and there is no solution.

On Your Own

7. (a) Two lines that do not intersect will be parallel, and parallel lines have the same slope. The equations will have the form $y = ax + b$ and $y = ax + c$.

(b) Two lines that intersect at the point $(1, -2)$ will both have a point-tester equation that looks like $y + 2 = m(x - 1)$. The two lines will have different slopes, so the m will be different.

8. (a) Kenji's first purchase was for 2 hats and 1 umbrella. His second purchase was for 2 hats and 3 umbrellas. So the second time, he bought 2 more umbrellas than the first, and the same number of hats. The difference between the purchases was 2 umbrellas, and the difference in cost was \$11.00. So 2 umbrellas must cost \$11.

(b) If 2 umbrellas cost \$11, 1 umbrella must cost \$5.50.

(c) 2 hats and an umbrella cost \$22, so 2 hats alone cost $\$22 - \$5.50 = \$16.50$. If 2 hats cost \$16.50, then 1 hat costs $\frac{1}{2}(\$16.50) = \$8.25$.

The reasoning used above is the very same reasoning you use when you use the elimination process. First, write two equations for the two purchases: if u is the price of an umbrella, and c is the price of a hat, then an equation for the first purchase would be $2h + u = 22$, and for the second purchase would be $2h + 3u = 33$. Subtract the first equation from the third equation to eliminate the h variable.

$$\begin{array}{rrcl} & 2h + 3u & = & 33 \\ (-) & 2h + u & = & 22 \\ \hline & 2u & = & 11 \\ & u & = & 5.50 \end{array}$$

On the left side, you found that the difference between the items purchased was 2 umbrellas. On the right, you found the difference in cost was \$11, just as before.

(d) One way to find a combination is to split 77 into a combination of 22 and 33, that is, $77 = 22 + 22 + 33$. Since $2h + u = 22$ and $2h + 3u = 33$, you can subsitute into the equation $77 = 22 + 22 + 33$:

$$\begin{aligned} 77 &= (2h + u) + (2h + u) + (2h + 3u) \\ 77 &= 6h + 5u \end{aligned}$$

Another way to find a solution is to notice that $2u = 11$ and $7(11) = 77$. Substitute the first into the second to get

$$\begin{aligned} 7(11) &= 77 \\ 7(2u) &= 77 \\ 14u &= 77 \end{aligned}$$

Finally, notice that $2h = 3u$. You can add $2h - 3u$ to any solution to find another solution, since $2h - 3u = 0$. Start with $14u = 77$ and add $2h - 3u = 0$, we get

$$\begin{aligned} 14u &= 77 \\ 14u + (2h - 3u) &= 77 + 0 \\ 11u + 2h &= 77 \end{aligned}$$

You can keep repeating to get a bunch of equations.

- $14u = 77$
- $2h + 11u = 77$
- $4h + 8u = 77$
- $6h + 5u = 77$
- $8h + 2u = 77$

9. Let w be the width of the pen, and let ℓ be the length. Then $2w = 3\ell + 1$. Also, $2w + 2\ell = 86$. Substitute to eliminate the $2w$ in the second equation to obtain $3\ell + 1 + 2\ell = 86$. Solving for ℓ, we get $5\ell = 85$, so $\ell = \frac{85}{5} = 17$. Therefore, $2w = 3(17) + 1 = 51 + 1 = 52$. Divide by 2 to obtain $w = \frac{52}{2} = 26$. The correct answer is **C**.

10. (a) The first equation is already solved for y, so you can use substitution.

$$\begin{aligned} 3x - 4y &= 41 \\ 3x - 4(-5x + 30) &= 41 \\ 3x + 20x - 120 &= 41 \\ 23x &= 161 \\ x &= 7 \end{aligned}$$

Plug that value back into the first equation to find y.

$$\begin{aligned} y &= -5x + 30 \\ y &= -5(7) + 30 \\ y &= -35 + 30 \\ y &= -5 \end{aligned}$$

So the intersection point is $(x, y) = (7, -5)$.

(b) Both equations are solved for y, so you can set the right sides equal to each other and solve for x.

$$\begin{aligned} 2x - 1 &= 9x + 6 \\ -7x &= 7 \\ x &= -1 \end{aligned}$$

Plug that value back into the first equation to find y.

$$\begin{aligned} y &= 2(-1) - 1 \\ y &= -2 - 2 \\ y &= -3 \end{aligned}$$

So the intersection point is $(x, y) = (-1, -3)$.

(c) The y-coefficients are the same, so elimination is the quickest way to solve the system.

$$\begin{array}{rrcl} & 3x + 2y & = & 7 \\ (-) & -2x + 2y & = & -2 \\ \hline & 5x & = & 9 \\ & x & = & \dfrac{9}{5} \end{array}$$

Plug that value back into the first equation to find y.

$$\begin{aligned} 3x + 2y &= 7 \\ 3\left(\frac{9}{5}\right) + 2y &= 7 \\ \frac{27}{5} + 2y &= 7 \end{aligned}$$

$$\begin{aligned} 2y &= 7 - \frac{27}{5} \\ 2y &= \frac{35}{5} - \frac{27}{5} \\ 2y &= \frac{8}{5} \\ y &= \frac{4}{5} \end{aligned}$$

So the intersection point is $(x, y) = \left(\frac{9}{5}, \frac{4}{5}\right)$.

(d) Elimination is again the way to go, since the coefficient of f is the same.

$$\begin{array}{rrcl} & 5e - 2f &=& 30 \\ (-) & 9e - 2f &=& 54 \\ \hline & -4e &=& -24 \\ & e &=& 6 \end{array}$$

Plug that value back into the first equation to find f.

$$\begin{aligned} 5e - 2f &= 30 \\ 5(6) - 2f &= 30 \\ 30 - 2f &= 30 \\ -2f &= 0 \\ f &= 0 \end{aligned}$$

So the intersection point is $(e, f) = (6, 0)$.

(e) Again, elimination is the best way to go, even though neither variable has the same coefficient in both equations. But notice that if you multiply the second equation by 9, you can eliminate the x terms.

$$\begin{array}{rrcl} & 27x + 5y &=& 30 \\ (+) & -27x - 18y &=& 9 \\ \hline & -13y &=& 39 \\ & y &=& -3 \end{array}$$

Plug that value into the second equation to find x.

$$\begin{aligned} -3x - 2y &= 1 \\ -3x - 2(-3) &= 1 \\ -3x + 6 &= 1 \\ -3x &= 1 - 6 \\ -3x &= -5 \\ x &= 5 \cdot \frac{1}{3} \\ x &= \frac{5}{3} \end{aligned}$$

So the intersection point is $(x, y) = \left(\frac{5}{3}, -3\right)$.

(f) When you have the equations in the form $ax + by = c$, elimination is usually the best way to go. Eliminate the k by multiplying the first equation by 3 and the second by 4.

$$\begin{array}{rrcl} & 15j - 12k &=& 57 \\ (-) & -28j - 12k &=& 88 \\ \hline & 43j &=& -31 \\ & j &=& -\frac{31}{43} \end{array}$$

Plug that value into the first equation to find k.

$$\begin{aligned} 5j - 4k &= 19 \\ 5\left(-\frac{31}{43}\right) - 4k &= 19 \\ -\frac{155}{43} - 4k &= 19 \\ -4k &= 19 + \frac{155}{43} \\ -4k &= \frac{817}{43} + \frac{155}{43} \\ -4k &= \frac{972}{43} \\ k &= -\frac{972}{43} \cdot \frac{1}{4} \\ k &= -\frac{243}{43} \end{aligned}$$

So the intersection point is $(j, k) = \left(-\frac{31}{43}, -\frac{243}{43}\right)$.

11. (a) Solve this system by multiplying the first equation by 2, the second by 3, and then adding:

$$\begin{array}{rrcl} & 4x + 6y &=& 10 \\ (+) & 9x - 6y &=& 42 \\ \hline & 13x &=& 52 \\ & x &=& 4 \end{array}$$

Plug 4 in for x in the first equation to find y.

$$\begin{aligned} 2x + 3y &= 5 \\ 2(4) + 3y &= 5 \\ 8 + 3y &= 5 \\ 3y &= -3 \\ y &= -1 \end{aligned}$$

So the solution is $(x, y) = (4, -1)$.

(b) The second equation in this system is the second equation from part (a) multiplied by 2, so the solution to this system is identical to the solution for part (a), $(x, y) = (4, -1)$. You could also solve by multiplying the first equation by 4, the second by 3, and then adding the equations.

(c) This system is similar to the system from part (a), with the only difference being that the y-coefficients are both the opposite of what they were in part (a). The result is that the solution for y will be the opposite of what it was in part (a).

You can test this guess by multiplying the first equation by 2, the second by 3, and then adding the equations:

$$\begin{array}{rrcl} & 4x - 6y &=& 10 \\ (+) & 9x + 6y &=& 42 \\ \hline & 13x &=& 52 \\ & x &=& 4 \end{array}$$

Now plug 4 in the first equation and solve for y.

$$\begin{aligned} 2x - 3y &= 5 \\ 2(4) - 3y &= 5 \end{aligned}$$

$$
\begin{aligned}
8 - 3y &= 5 \\
-3y &= -3 \\
y &= 1
\end{aligned}
$$

So the solution is, as expected, $(x, y) = (4, 1)$.

(d) Solve this system by multiplying the first equation by 3 and then subtracting:

$$
\begin{array}{rl}
 & 6x + 9y = 15 \\
(-) & 6x + 9y = 18 \\
\hline
 & 0x + 0y = -3 \\
 & 0 \neq -3
\end{array}
$$

Whoops! You got an equation that can never be true, so there is no intersection. The lines are parallel, and their slopes are equal.

12. (a) To solve a system with three equations and two unknowns, take each possible combination of two lines and solve that smaller system.

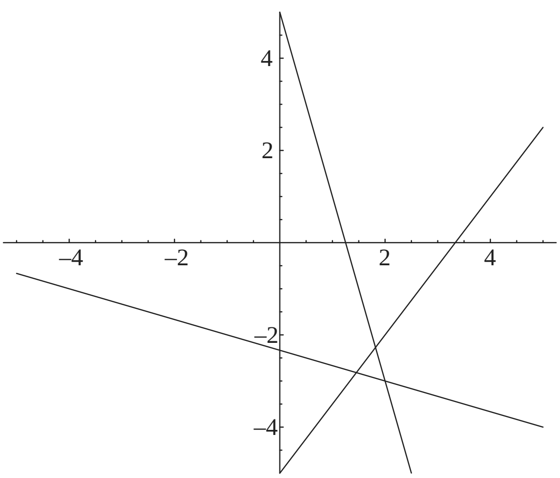
Find the intersection point between the first two equations by multiplying the second equation by 2 and adding the two together.

$$
\begin{array}{rl}
 & 3x - 2y = 10 \\
(+) & 8x + 2y = 10 \\
\hline
 & 11x = 20 \\
 & x = \dfrac{20}{11}
\end{array}
$$

Before you go ahead and find the y-coordinate of that point, solve the system that includes the second and third equations first. Eliminate y again and solve for x. If you get a different number, then the two points cannot be the same, and there won't be a single solution for all three lines. Multiply the second equation by 3 and subtracting the two equations.

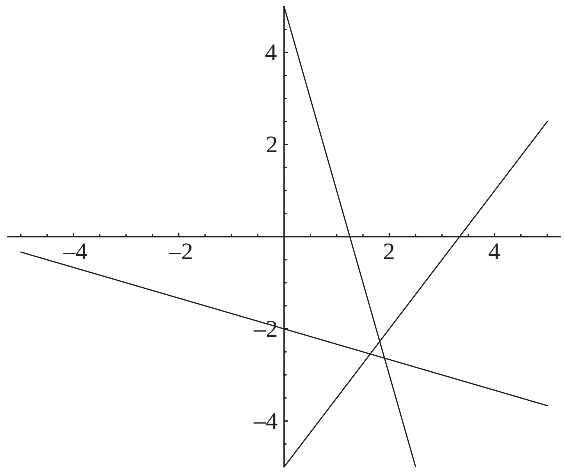
$$
\begin{array}{rl}
 & 12x + 3y = 15 \\
(-) & x + 3y = -7 \\
\hline
 & 3x = 22 \\
 & x = \dfrac{22}{3}
\end{array}
$$

The value for x is different, so the three lines cannot intersect in one single point. Take a look at the graph of the three lines:



Notice that there are three intersection points, where each pair of lines intersects. There is not a single point that is on all three lines.

(b) In part (a), you saw that three lines may not intersect in a single point (in fact, they usually will not). You may be suspicious as to whether these three will intersect in a single point, so looking at the graph first may help.

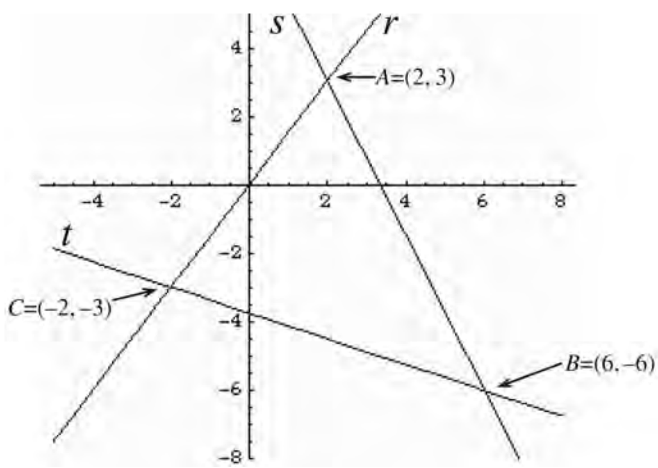


The intersection points are closer, but it still looks like there are three separate intersection points, making a triangle. You can make sure there is not a single intersection point by following the procedure from part (a). The first two lines in this system are the same as in part (a), so you already know the x-coordinate of that intersection point, $x = \frac{20}{11}$. So find the x value from the system that includes the second and third equations by multiplying the second by 3 and subtracting.

$$
\begin{array}{rl}
 & 12x + 3y = 15 \\
(-) & x + 3y = -6 \\
\hline
 & 11x = 21 \\
 & x = \dfrac{21}{11}
\end{array}
$$

Since $\frac{21}{11} \neq \frac{20}{11}$, the two systems do not have the same x value, so there isn't a single point of intersection for the three lines.

13. If you graph the lines, you'll see that their intersections form a triangle.



Take the lines, two at a time, and find the intersection points. All three equations are solved for y, so the easiest way to solve is to use substitution (setting the right sides equal to each other). Start with lines r and s. Call that intersection A.

$$\begin{aligned} \frac{3}{2}x &= -\frac{9}{4}x + \frac{15}{2} && \text{Add } \tfrac{9}{4}x \text{ to both sides.} \\ \frac{3}{2}x + \frac{9}{4}x &= \frac{15}{2} && \text{Multiply both sides by 4.} \\ 6x + 9x &= 30 \\ 15x &= 30 \\ x &= 2 \end{aligned}$$

If you love fractions, you could skip that second step and continue. But most people find it easier to work with integers instead of fractions, so when you have the option, go for it. Anyway, plug 2 in for x in the equation for r.

$$\begin{aligned} y &= \frac{3}{2}x \\ y &= \frac{3}{2}(2) \\ y &= 3 \end{aligned}$$

So the intersection point of r and s is $A = (2, 3)$.
Next, find the intersection of s and t, and call that intersection B.

$$\begin{aligned} -\frac{9}{4}x + \frac{15}{2} &= -\frac{3}{8}x - \frac{15}{4} && \text{Multiply both sides by 8.} \\ -18x + 60 &= -3x - 30 \\ -15x &= -90 \\ x &= 6 \end{aligned}$$

Plug 6 in for x in the equation for s.

$$\begin{aligned} y &= -\frac{9}{4}x + \frac{15}{2} \\ y &= -\frac{9}{4}(6) + \frac{15}{2} \\ y &= -\frac{27}{2} + \frac{15}{2} \\ y &= -\frac{1}{2}2 \\ y &= -6 \end{aligned}$$

The intersection point of s and t is $B = (6, -6)$.
Finally, find the intersection of r and t, and call the intersection C.

$$\begin{aligned} \frac{3}{2}x &= -\frac{3}{8}x - \frac{15}{4} && \text{Multiply both sides by 8.} \\ 12x &= -3x - 30 \\ 15x &= -30 \\ x &= -2 \end{aligned}$$

Plug 2 in for x in the equation for r.

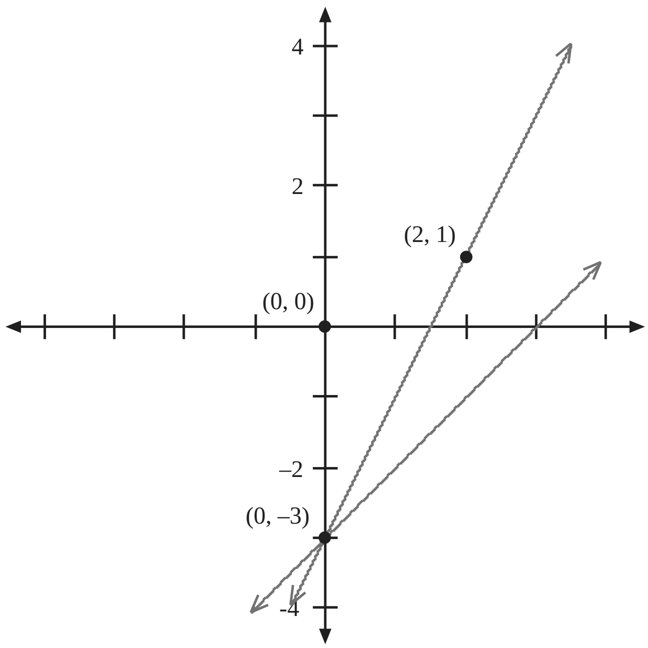
$$\begin{aligned} y &= \frac{3}{2}x \\ y &= \frac{3}{2}(-2) \\ y &= -3 \end{aligned}$$

The intersection point of r and t is $C = (-2, -3)$.
So, the vertices of the triangle are: $A = (2, 3)$, $B = (6, -6)$, and $C = (-2, -3)$.

14. The pair (1, 1) is a solution to every equation in the list, so it is a solution to any system of equations composed of two equations from the list.

4C MATHEMATICAL REFLECTIONS

1. (a) Answers will vary. For example, looking at the graph, $(0, 0)$ is not on either line.
(b) Answers will vary. For example, $(2, 1)$ is on $y = 2x - 3$ because $1 = 2(2) - 3$, but is not on $x - y = 3$ because $2 - 1 \neq 3$.
(c) Looking at the graph, the only point on both lines is $(0, -3)$.



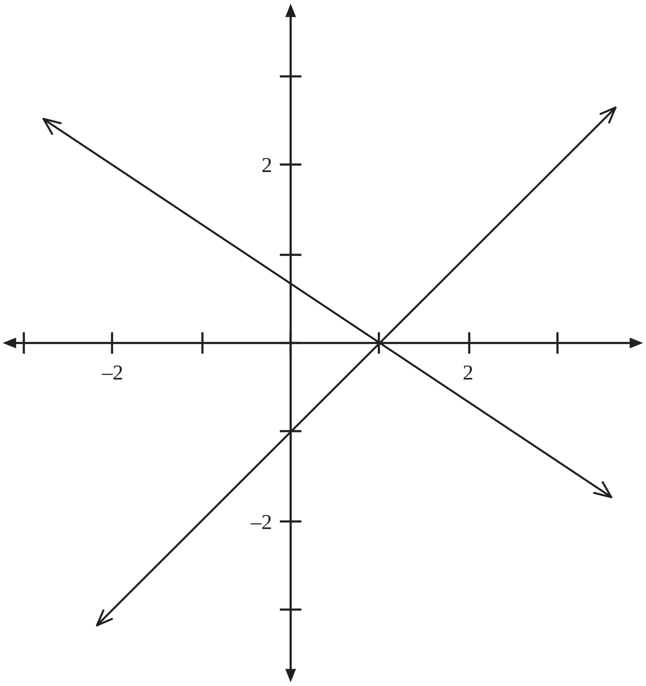
2. Answers will vary. Use a point-tester equation with (x, y) and $(1, -3)$. Choose two different values for the slope, m. If $m = 2$,

$$\begin{aligned} \frac{y - (-3)}{x - 1} &= 2 \\ y + 3 &= 2(x - 1) \\ y + 3 &= 2x - 2 \\ y &= 2x - 5 \\ 5 &= 2x - y \end{aligned}$$

If $m = -2$,

$$\begin{aligned} \frac{y - (-3)}{x - 1} &= -2 \\ y + 3 &= -2(x - 1) \\ y + 3 &= -2x + 2 \\ 2x + y &= -1 \end{aligned}$$

3. (a)

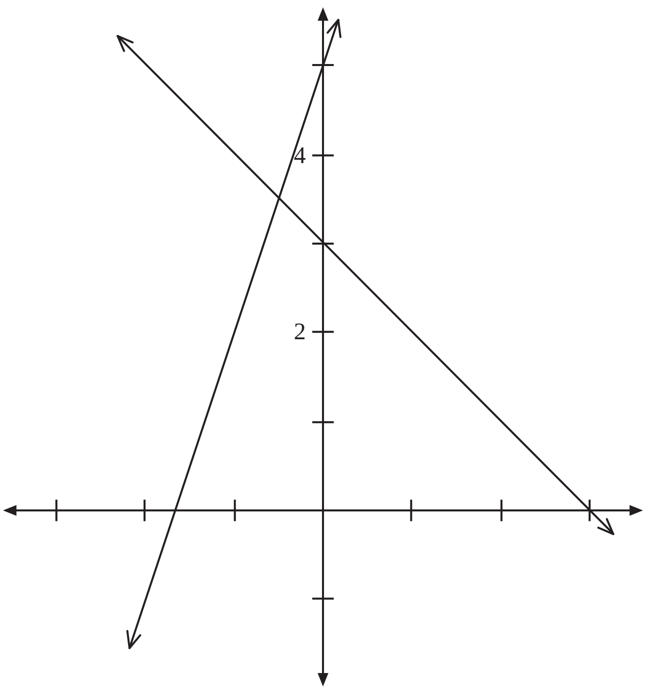


Since the second equation is solved for y, substitute it into the first equation:

$$\begin{aligned} 2x + 3(x-1) &= 2 \\ 2x + 3x - 3 &= 2 \\ 5x - 3 &= 2 \\ 5x &= 5 \\ x &= 1 \end{aligned}$$

Substitute into either equation to find y: $y = x - 1 = 1 - 1 = 0$. So the point is $(1, 0)$.

(b)



Solve one of the equations for either x or y:

$$\begin{aligned} x + y &= 3 \\ y &= 3 - x \end{aligned}$$

Substitute into the first equation:

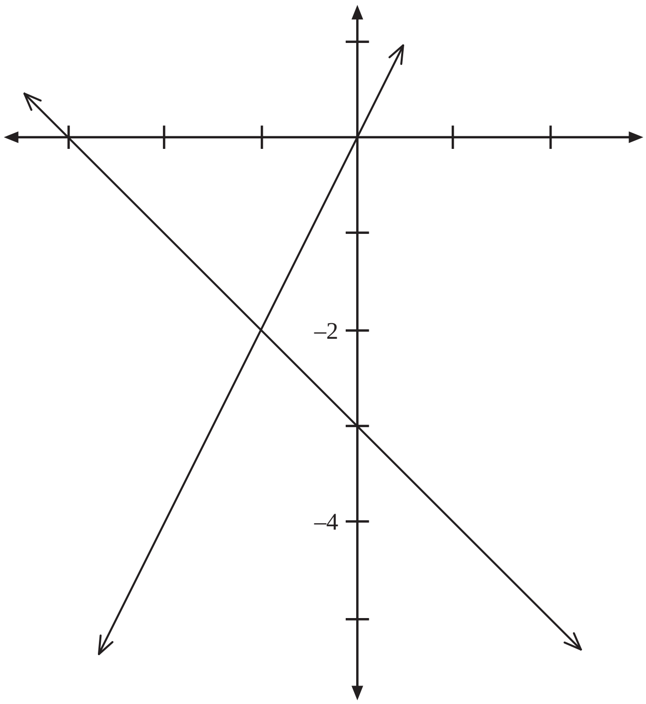
$$\begin{aligned} y - 2 &= 3(x+1) \\ (3-x) - 2 &= 3(x+1) \\ 1 - x &= 3x + 3 \\ 1 - x - 3x &= 3x - 3x + 3 \\ 1 - 4x &= 3 \\ -4x &= 2 \\ \frac{-4x}{-4} &= \frac{2}{-4} \\ x &= -\frac{1}{2} \end{aligned}$$

Substitute to find y:

$$x + y = 3 \Rightarrow -\frac{1}{2} + y = 3 \Rightarrow y = 3 + \frac{1}{2} = \frac{7}{2}$$

The point is $\left(-\frac{1}{2}, \frac{7}{2}\right)$.

(c)



Since both equations are solved for y, set them equal to each other.

$$\begin{aligned} 2x &= -x - 3 \\ 3x &= -3 \\ x &= -1 \\ y = 2x &= 2(-1) = -2 \end{aligned}$$

The point is $(-1, -2)$.

4. (a) Answers will vary. Find the slope of $2x - y = 4$. If $x = 1$, $2(1) - y = 4 \Rightarrow -y = 2 \Rightarrow y = -2$. If

$x = 2$, $2(2) - y = 4 \Rightarrow 4 - y = 4 \Rightarrow y = 0$. So, two points are $(1, -2)$ and $(2, 0)$. The slope of the line is

$$\frac{-2-0}{1-2} = \frac{-2}{-1} = 2$$

A parallel line will have the same slope, but no common points. Choose any point not on $2x - y = 4$, say $(4, 2)$, and use the point-tester equation with (x, y) and $(4, 2)$:

$$\begin{aligned}\frac{y-2}{x-4} &= 2\\ y-2 &= 2(x-4)\\ y-2 &= 2x-8\\ 6 &= 2x-y\\ 2x-y &= 6\end{aligned}$$

(b) Answers will vary. Choose a slope that is not equal to 2, say -3. Use the point-tester equation.

$$\begin{aligned}\frac{y-(-2)}{x-1} &= -3\\ y+2 &= -3(x-1)\\ y+2 &= -3x+3\\ 3x+y &= 1\end{aligned}$$

5. (a) Since $-2y + 2y = 0y = 0$, add the two equations to eliminate y:

$$\begin{array}{r} x-2y=5\\ (+)3x+2y=-1\\ \hline 4x+0y=4\\ x=1\end{array}$$

Substitute to find y:

$$\begin{aligned}x-2y &= 5\\ 1-2y &= 5\\ -2y &= 4\\ y &= -2\end{aligned}$$

The solution is $(1, -2)$.

(b) Multiply the second equation by 3 because $-3y + 3y = 0y$. Then add the two equations to eliminate y.

$$\begin{aligned}2x-3y &= 15\\ x+y &= 10\end{aligned}$$

$$\begin{array}{r} 2x-3y=15\\ (+)3x+3y=30\\ \hline 5x+0y=45\\ x=9\end{array}$$

Substitute to find y:

$$\begin{aligned}x+y &= 10\\ 9+y &= 10\\ y &= 1\end{aligned}$$

The solution is $(9, 1)$.

(c) Multiply the first equation by 2 and the second equation by -1, and add the two equations together.

$$\begin{aligned}4x+3y &= 12\\ 8x+6y &= 8\end{aligned}$$

$$\begin{array}{r} 8x+6y=24\\ (+)-8x-6y=-8\\ \hline 0\neq 16\end{array}$$

There is no solution.

(d) Multiply the first equation by 2 and the second equation by 3, then add the two equations to eliminate y:

$$\begin{aligned}2x+3y &= 1\\ 4x-2y &= 10\end{aligned}$$

$$\begin{array}{r} 4x+6y=2\\ 12x-6y=30\\ \hline 16x+0y=32\\ x=2\end{array}$$

Substitute to find y:

$$\begin{aligned}2x+3y &= 1\\ 2(2)+3y &= 1\\ 4+3y &= 1\\ 3y &= -3, \text{ or } y=-1\end{aligned}$$

The solution is $(2, -1)$.

6. You can find the intersection of two lines by finding the values of the variables that satisfy both equations. You can find these values algebraically by solving the system of equations. Use methods such as substitution or elimination.

7. Two lines will intersect if they do not have the same slope. So, find the slope of each line to determine if the lines are parallel or not.

8. Two lines are parallel if they have the same slope. The slope of the line from the second equation is $\frac{3}{4}$. You can find the slope of the first equation by solving for y:

$$\begin{aligned}3x-4y &= 16\\ -4y &= -3x+16\\ y &= \frac{3}{4}x-4\end{aligned}$$

So the slope of line from the first equation is also $\frac{3}{4}$. Since the two lines have the same slope, they are parallel.

INVESTIGATION 4D APPLICATIONS OF LINES

4.13 Getting Started

For You to Explore

1. (a) Replace x with the number 5:

$$\begin{aligned} 4x - 7 &= -2x + 9 \\ 4(5) - 7 &\stackrel{?}{=} -2(5) + 9 \\ 20 - 7 &\stackrel{?}{=} -10 + 9 \\ 13 &\neq -1 \end{aligned}$$

So the equation is false when $x = 5$.

(b) Solve the equation for x:

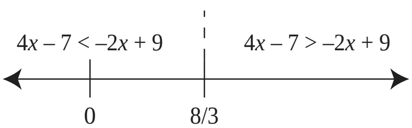
$$\begin{aligned} 4x - 7 &= -2x + 9 \\ 4x - 7 + 2x &= -2x + 9 + 2x \\ 6x - 7 &= 9 \\ 6x - 7 + 7 &= 9 + 7 \\ 6x &= 16 \\ x &= \frac{16}{6} = \frac{8}{3} \end{aligned}$$

(c) Any number other than $\frac{8}{3}$ will make the equation false.

(d) There are many possible answers. Five are 0, 1, $\frac{5}{2}$, -5, and -9.

(e) There are many possible answers. Five are 3, 4, 10, 100, and 1234.

(f) All the numbers that satisfy $4x - 7 < -2x + 9$ are less than $\frac{8}{3}$. At the same time, all the numbers that satisfy $4x - 7 > -2x + 9$ are greater than $\frac{8}{3}$. Here is a graph of where those numbers are located:



2. (a) Tax is calculated as $5\% \times \$49.99 = \$2.4995$, which rounds up to \$2.50. The total price is $\$49.99 + \$2.50 = \$52.49$.

(b) No, since Justin will have $\$10 + \$6 \cdot 7 = \$52$ if he trades in all his games, which is not enough to pay the \$52.49 for the game.

(c) Yes, since Jason would have $\$20 + \$6 \cdot 15 = \$110$ if he trades in all his games, which is more than enough to pay for the game.

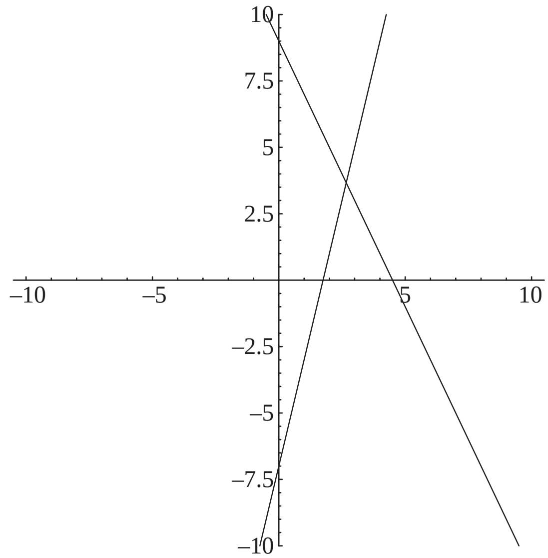
(d) If n is the number of games Jason trades, he will have $20 + 6n$ dollars to trade. For this to be enough, solve the algebra problem:

$$\begin{aligned} 20 + 6n &= 52.49 \\ 20 + 6n - 20 &= 52.49 - 20 \\ 6n &= 32.49 \\ n &= \frac{32.49}{6} \approx 5.415 \end{aligned}$$

Jason cannot buy 0.415 games, so he needs to round his answer to an integer. Since 5 is not enough, he needs to trade at least 6 old games.

(e) Jason has ten options: trade 6 games, 7 games, 8 games, . . . , all 15 games. One way to say this quickly is that Jason can trade any number of games between 6 and 15, inclusive.

3. The following figure shows the two graphs.



To find the intersection, use substitution. The x-coordinate of the intersection will be the value that solves $4x - 7 = -2x + 9$. This equation is solved in Exercise 1, and the x-value is $\frac{8}{3}$. Using this value for x gives the y-value of

$$4 \cdot \frac{8}{3} - 7 = \frac{11}{3}$$

4. (a) When the two graphs intersect, they have the same x and y value (since both graphs share the same point). From the figure, you can see that the two graphs intersect in 2 places: $(3, 4)$ and $(-2, -1)$.

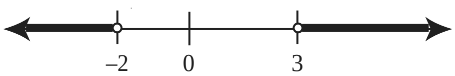
Both equations that make up the graphs have just y on the left side of the equation. So, since the two graphs have the same y value at these points, the expressions on the right side must also be equal at these points. If you set those two expressions equal to each other, you get the equation $x^2 - 5 = x + 1$, the equation you are asked to solve! So the two solutions must be the x-coordinates of the intersections of the given graphs.

Test that 3 and -2 are each a solution by plugging them into the equation

$$\begin{aligned} x^2 - 5 &= x + 1 \\ 3^2 - 5 &\stackrel{?}{=} 3 + 1 \\ 4 &= 4 \quad \checkmark \end{aligned}$$

$$(-2)^2 - 5 \stackrel{?}{=} (-2) + 1$$
$$-1 = -1 \quad \checkmark$$

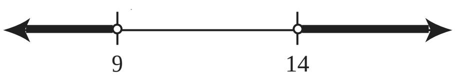
(b) According to the graphs, the graph of $y = x^2 - 5$ is higher than the graph of $y = x + 1$ when x is greater than 3 or less than -2, so those are the intervals where the inequality $x^2 - 5 > x + 1$ is true:



5. (a) These equations are identical to the ones in Exercise 4, except x has been replaced by $(x - 11)$. This means the two graphs are each moved 11 units to the right, and so the intersections will also move 11 units to the right.

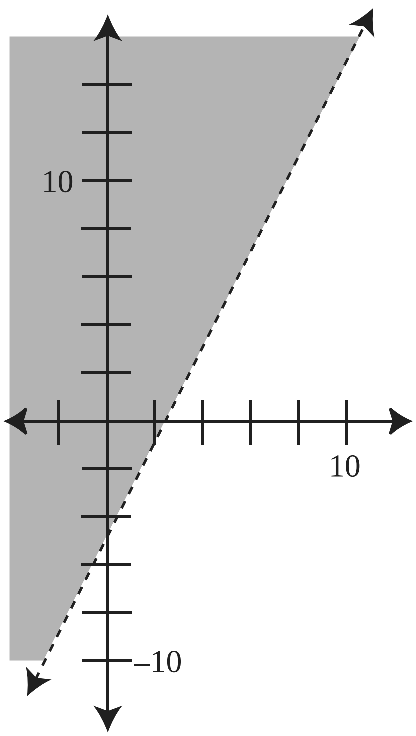
In Exercise 4, you found the intersections were at $x = 3$ and $x = -2$, so add 11 to each of these to find the two values of x: 14 and 9.

(b) The answer to part (b) is just like the answer to part (b) of Exercise 4, only moved 11 units to the right.

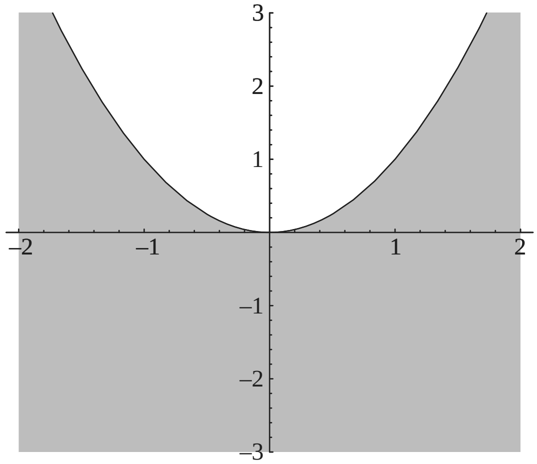


6. (a) Answers may vary. Sample: $(0, 0)$, $(0, -3)$, $(2, 5)$, $(-2, -1)$, $(-5, -4)$, $(4, 7)$, and $(10, 35)$.

(b) The graph will be every point that is above the line $y = 2x - 5$. The conventional way to label this is to use a dotted line, which indicates that the line is not part of the solution, and shade everything above it. Remember: the line is not part of the solution, but every point above the line is.



7. The graph includes the curve $y = x^2$ as part of the solution, since any point that makes $y = x^2$ true also makes $y \le x^2$ true. All the points that fall below that curve also makes the inequality true.



On Your Own

8. (a) Replace x with the number 7:

$$(x - 3)^2 + 5 = 41$$
$$(7 - 3)^2 + 5 \stackrel{?}{=} 41$$
$$21 \ne 41$$

So the statement is false.

(b) You show that the equation is true when $x = 3$ by replacing x with -3 and simplifying:

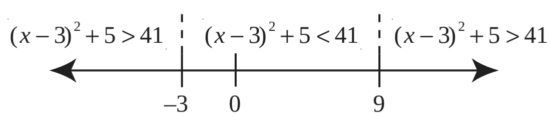
$$(-3 - 3)^2 + 5 \stackrel{?}{=} 41$$
$$(-6)^2 + 5 \stackrel{?}{=} 41$$
$$36 + 5 \stackrel{?}{=} 41$$
$$41 = 41$$

(c) The solution can be found by backtracking. If $(x - 3)^2 + 5$ makes 41, then $(x - 3)^2$ must be 36. If $(x - 3)^2$ is 36, then $(x - 3)$ has to either be 6 or -6. When $x - 3$ is 6, then x is 9. When $x - 3$ is -6, then x is -3 (the other given solution).

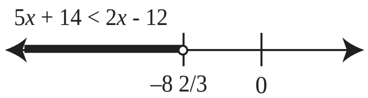
(d) There are many possible answers. Five are 4, 6, 0, -1, and $\frac{1}{2}$.

(e) There are many possible answers. Five are 100, -11, 15, -3.1, and 1234.

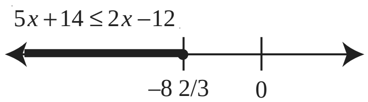
(f) All the numbers that satisfy $(x - 3)^2 + 5 < 41$ are between the two solutions -3 and 9. All the numbers that satisfy $(x - 3)^2 + 5 > 41$ are either greater than 9 or less than -3. Here is a number line with that information:



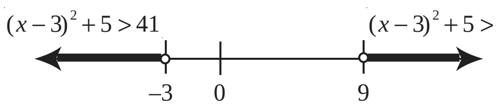
9. (a) The solution set to $5x + 14 < 2x - 12$ is all numbers less than $-8\frac{2}{3}$:



(b) The solution set to $5x + 14 \le 2x - 12$ is all numbers less than or equal to $-8\frac{2}{3}$. This is identical to the previous exercise, except it uses a closed circle since $-8\frac{2}{3}$ is now a solution:



(c) The solution set to $(x - 3)^2 + 5 > 41$ is all numbers larger than 9 *or* smaller than -3:



10. (a) There are three intersection points.

(b) The intersections are $(-2, -2)$, $(0, 0)$, and $(2, 2)$.

(c) When the two graphs intersect, they have the same x and y value (since both graphs share the same point). Both equations that make up the graphs have just y on the left side of the equation. So, since the two graphs have the same y value at these points, the expressions on the right side must also be equal at these points. If you set those two expressions equal to each other, you get the equation $x^3 - 3x = x$, the equation you are asked to solve! So the two solutions must be the x-coordinates of the intersections of the given graphs: $x = -2$, $x = 0$, and $x = 2$.

(d) The value of x is larger than the value of $x^3 - 3x$ whenever x is between 0 and 2, and whenever x is less than -2. This can be seen from the graph, and the intersection points you found determine when there is a change.

11. (a) The equation $3a - 16 = 23$ is true when $a = 13$. Testing numbers larger than 13 or smaller than 13 suggests that the inequality is true whenever a is greater than 13.

(b) As before, the equation $3b - 16 = 23$ is true when $b = 13$. The only difference between this inequality and the one in part (a) is that b can also equal 13, so this inequality is true when a is at least (or greater than or equal to) 13.

(c) The equation $|d - 4| = 3$ has two solutions, the values of d that are 3 units away from 4 on the number line. Those values are $d = 1$ and $d = 7$. For $|d - 4|$ to be *less than* 3, the value of d must be less than 3 units away on the number line. That means d must be between 1 and 7 (that is, d is larger than 1 and smaller than 7).

(d) The only difference between this inequality and the one in part (c) is that the values $h = 1$ and $h = 7$ are part of the solution. So h must be at least 1 and at most 7 (that is, between 1 and 7 inclusive).

(e) The equation $(d - 4)^2 = 9$ has two solutions. The value of $(d - 4)$ can either be 3 or -3, giving $d = 1$ and $d = 7$ as the two solutions. Testing values can help find that d must be between 1 and 7 to make the inequality true, as does knowing that to have a number's square be less than 9, the number must be between -3 and 3.

Maintain Your Skills

12. (a) If $n = 10$, then by the basic moves, you know that $2n = 20$. So the turning point will be at 20. To make sure whether the correct answer is greater than or less than, try a value. If $n = 12$, then it is true that n is greater than 10. It is also true that $2n$, which equals 24, is greater than 20. So the value of $2n$ is greater than 20.

(b) Again, if you use the basic rules, if $n = 10$, then $n - 20 = 10 - 20$, or $n - 20 = -10$. Use 11 again for n, and you see that $12 - 20$, which equals -8, is indeed greater than -10. So the value of $n - 20$ is greater than -10.

(c) If $n = 10$, then $-n = -10$. You might think that the value of n must then be greater than -10. But first, to make sure, try 12. 12 is (still!) greater than 10, but -12 is *not* greater than -10, it's less than -10. Try a few more, and you'll see that the value of $-n$ is *less than* -10.

(d) $n = 10$ means that $\frac{n}{2} = \frac{10}{2}$, or $\frac{n}{2} = 5$. Plug in 12 and see that $\frac{12}{2}$, which is 6, is greater than 5. So the value of $\frac{n}{2}$ is greater than 5.

(e) Okay, if $n = 10$, then by the basic moves,

$$\begin{aligned} n &= 10 \\ -3n &= -3 \cdot 10 \\ -3n &= -30 \\ -3n + 7 &= -30 + 7 \\ -3n + 7 &= -23 \end{aligned}$$

So -23 is the turning point. Again, try 12 and see what happens: $-3(12) + 7 = -36 + 7 = -29$, which is less than -23. So, the value of $-3n + 7$ is *less than* -23.

13. You can solve each part using substitution. But do you see a pattern? You can find the rest of the answers once you have solved part (a) without going through all the steps again.

(a) Solve by substitution:

$$\begin{aligned} 3x - 5 &= -2x + 10 \\ 3x - 5 + 2x &= -2x + 10 + 2x \\ 5x - 5 &= 10 \\ 5x &= 15 \\ x &= \frac{15}{5} = 3 \end{aligned}$$

Then, plug in $x = 3$ to find y in either equation:

$$y = 3(3) - 5 = 9 - 5 = 4$$

(b) Solve by substitution:

$$\begin{aligned} 3x - 8 &= -2x + 7 \\ 3x - 8 + 2x &= -2x + 7 + 2x \\ 5x - 8 &= 7 \\ 5x &= 15 \\ x &= \frac{15}{5} = 3 \end{aligned}$$

Then, plug in $x = 3$ to find y in either equation:

$$y = 3(3) - 8 = 1$$

(c) Solve by substitution:

$$\begin{aligned} 3x + 5 &= -2x + 20 \\ 3x + 5 + 2x &= -2x + 20 + 2x \\ 5x + 5 &= 20 \\ 5x &= 15 \\ x &= \frac{15}{5} = 3 \end{aligned}$$

Then, plug in $x = 3$ to find y in either equation:

$$y = 3(3) + 5 = 14$$

(d) Solve by substitution:

$$\begin{aligned} 3(x - 4) - 5 &= -2(x - 4) + 10 \\ 3x - 12 - 5 &= -2x + 8 + 10 \\ 3x - 17 &= -2x + 18 \\ 3x - 17 + 2x &= -2x + 18 + 2x \\ 5x - 17 &= 18 \\ 5x &= 35 \\ x &= \frac{35}{5} = 7 \end{aligned}$$

Then, plug in $x = 7$ to find y in either equation:

$$y = 3(7 - 4) - 5 = 3(3) - 5 = 4$$

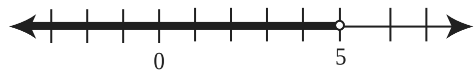
(e) The graphs of the first equation in each pair are all parallel, with a slope of 3. The graphs of the second equation in each pair are also all parallel, with a slope of -2.

4.14 Inequalities With One Variable

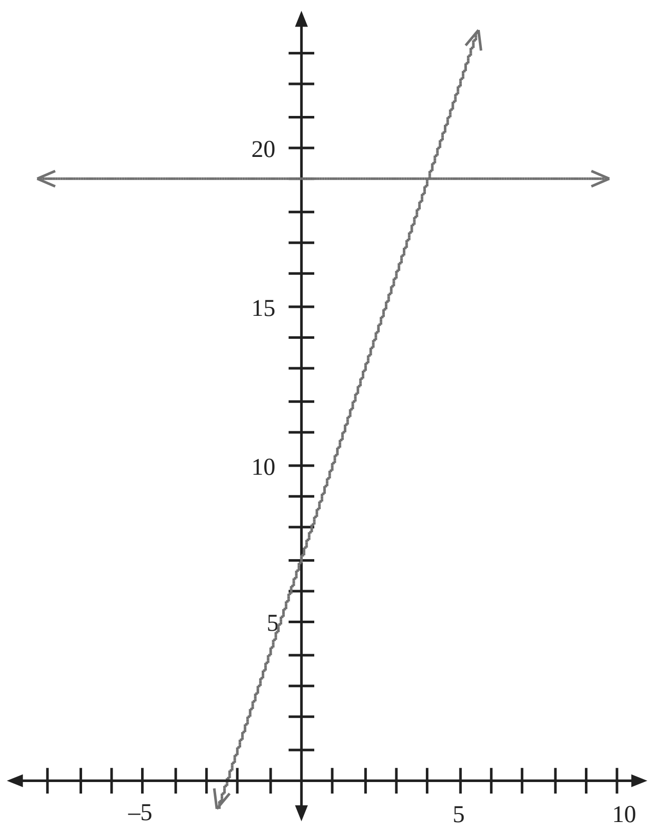
Check Your Understanding

1. There are a number of methods you can use to solve inequalities, as presented in the lesson. Each of these solutions will highlight a different method. There is no “best” or “correct” method—use the method that makes the most sense to you.

(a) This inequality will be solved using the idea of cutoff points. First, solve the equation $5x = 25$ to get $x = 5$, so 5 is a solution to the corresponding equation, and so the number line is broken into two regions with 5 as the cutoff point. Test a convenient point on one side of the cutoff point, such as 0: $5(0) < 25$ is true, so the region with 0 is shaded. Pick a number in the other region, such as 10, and you get $5(10) < 25$, which is false, so that region is not shaded. Thus, any number x such that $x < 5$ will work. Use an open circle for 5, since 5 is not part of the solution set.



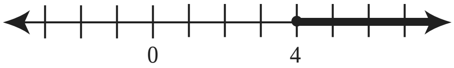
(b) This inequality will be solved using the two-graphs method. Graph the two equations $y = 3x + 7$ and $y = 19$ on the same coordinate axes.



From the graph, you can see that the y-height of the line $y = 3x + 7$ is greater than the line $y = 19$ everywhere to the right of the intersection point. That intersection looks to be about $(4, 19)$; solve the corresponding equation to be sure.

$$\begin{aligned} 3x + 7 &= 19 \\ 3x &= 19 - 7 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

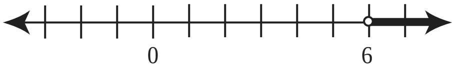
So indeed, the intersection is at $x = 4$, so $3x + 7 \geq 19$ whenever $x \geq 4$. Use a closed circle for 4, since 4 is part of the solution set.



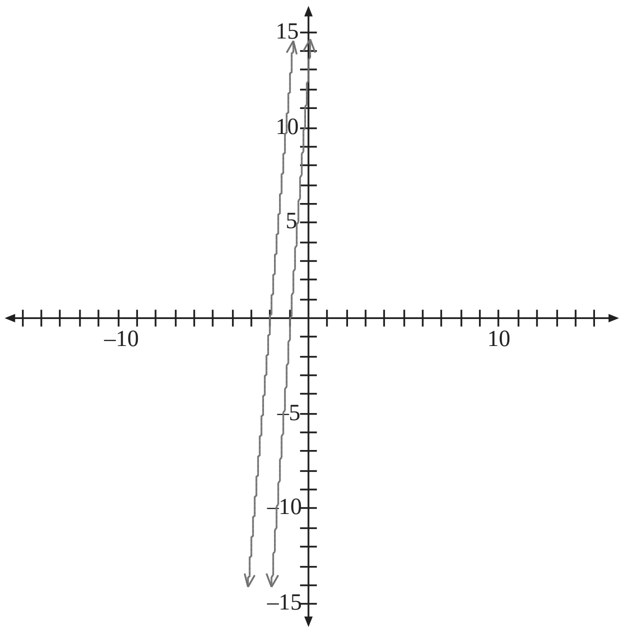
(c) Using the cutoff point method, solve the equation $4x - 9 = 2x + 3$.

$$\begin{aligned} 4x - 9 &= 2x + 3 \\ 4x - 2x &= 9 + 3 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

6 is the cutoff point. If you plug in 0 for x, you get $-9 > 3$, which is false. Since 0 is not part of the solution, that region is not shaded. Then plug in 10 for x to get $31 > 23$, which is true, so that region is shaded. Thus, any number x such that $x > 6$ will work. Use an open circle since 6 is not part of the solution set.



(d) Use the two-graphs method. Graph the two equations $y = 14x + 13$ and $y = 6(2x + 4)$ together.



Judging from this graph, it looks like the two lines are parallel. Well, if that is the case, then there will not be any intersection point. So, try to find an intersection point by solving the equation $14x + 13 = 6(2x + 4)$.

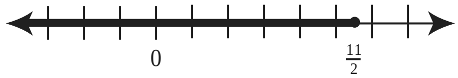
$$\begin{aligned} 14x + 13 &= 6(2x + 4) \\ 14x + 13 &= 12x + 24 \\ 14x - 12x &= 24 - 13 \\ 2x &= 11 \\ x &= \frac{11}{2} \end{aligned}$$

So, there is an intersection point. The y-coordinate is

$$\begin{aligned} y &= 14x + 13 \\ y &= 14\left(\frac{11}{2}\right) + 13 \\ y &= 77 + 13 \\ y &= 90 \end{aligned}$$

If you are graphing your equations on your calculator, you can adjust your window so that the point $\left(\frac{11}{2}, 90\right)$ is in the center. Even so, the two lines have such a similar slope, it can be difficult to keep track of which line is which. So for this inequality, the cutoff point method might work best.

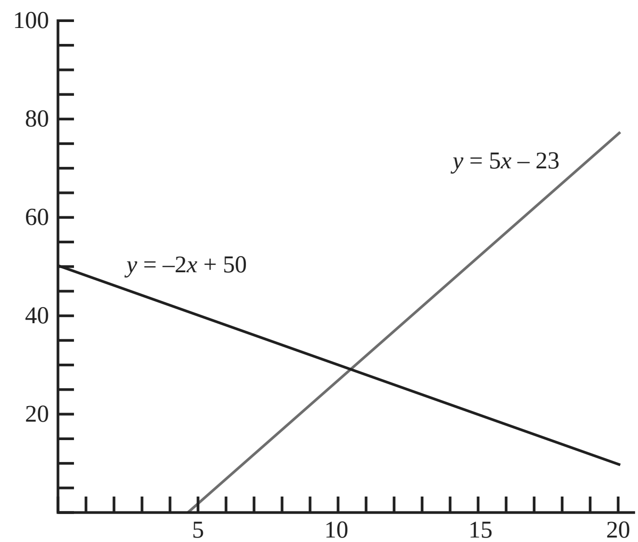
Test the region less than $\frac{11}{2}$ by plugging in 0 for x. The result is $13 \leq 24$, which is true, so 0 is part of the solution set and that region is shaded. Next, plug in 10 for x to get $153 \leq 144$, which is false, so that region is not shaded. Thus, any number x such that $x \leq \frac{11}{2}$ will work. Use a closed circle since $\frac{11}{2}$ is part of the solution.



2. (a) First, solve the equality

$$\begin{aligned} 5x - 23 &= -2x + 50 \\ 5x - 23 + 2x &= -2x + 50 + 2x \\ 7x - 23 &= 50 \\ 7x &= 73 \\ x &= \frac{73}{7} = 10\frac{3}{7} \end{aligned}$$

The figure below shows the two lines $y = 5x - 23$ and $y = -2x + 50$ on a graphing calculator, with window ranges $0 \leq x \leq 20$ and $0 \leq y \leq 100$:

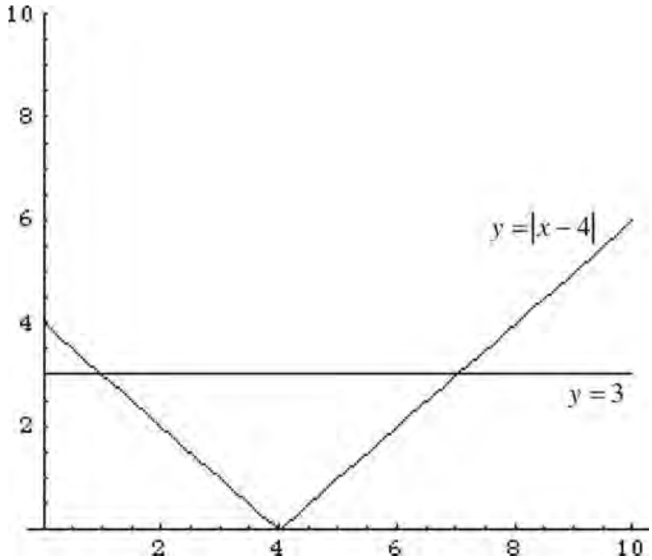


The graphs show that $5x - 23 > -2x + 50$ only when x is larger than $10\frac{3}{7}$.

(b) Solve the equality

$$|x-4| = 3$$
$$x-4 = \pm 3$$
$$x = 4 \pm 3 = 1 \text{ or } 7$$

Next, graph the equations $y = |x-4|$ and $y = 3$ on your graphing calculator, using window ranges $0 \le x \le 10$ and $0 \le y \le 10$:

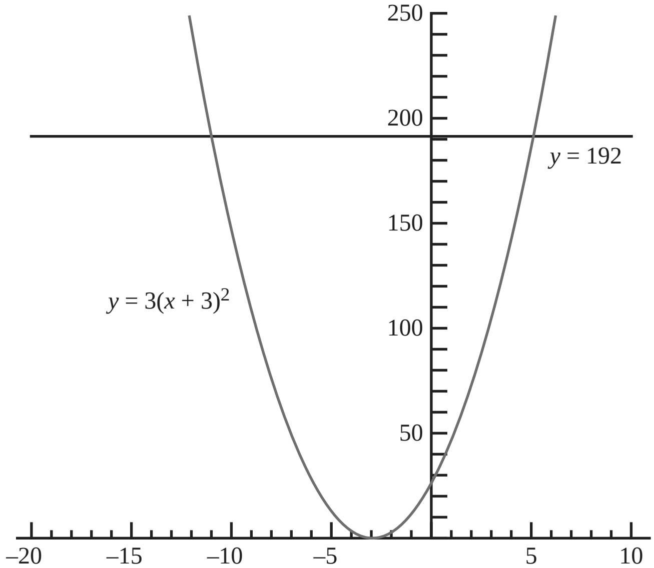


The graphs show that $|x-4| < 3$ only when x is between 1 and 7.

(c) Solve the equality

$$3(x+3)^2 = 192$$
$$(x+3)^2 = 64$$
$$\sqrt{(x+3)^2} = \sqrt{64}$$
$$x+3 = \pm 8$$
$$x = -3 \pm 8 = -11 \text{ or } 5$$

Graph the equations $y = 3(x+3)^2$ and $y = 192$ on your graphing calculator, using window ranges $-20 \le x \le 10$ and $0 \le y \le 250$:

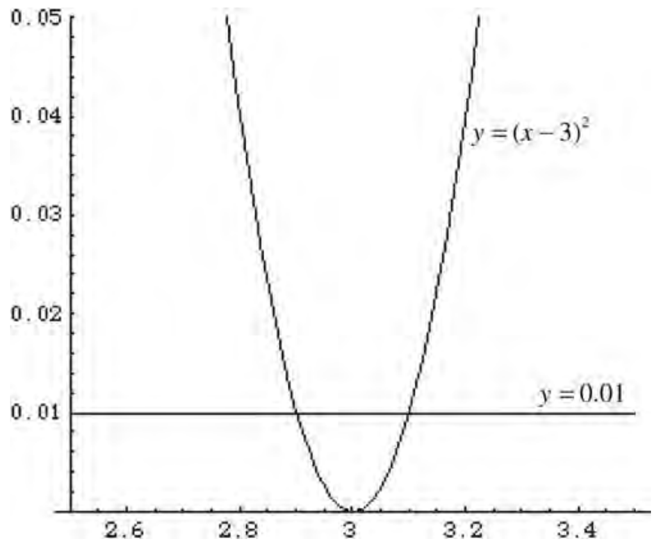


The graphs show that $3(x+3)^2 > 192$ only when x is less than -11 and when x is greater than 5.

(d) Solve the equality

$$(x-3)^2 = 0.01$$
$$\sqrt{(x-3)^2} = \sqrt{0.01}$$
$$x-3 = \pm 0.1$$
$$x = 3 \pm 0.1 = 2.9 \text{ or } 3.1$$

Next, graph the equations $y = (x-3)^2$ and $y = 0.01$ on your graphing calculator, with window ranges $2.5 \le x \le 3.5$ and $0 \le y \le 0.05$:

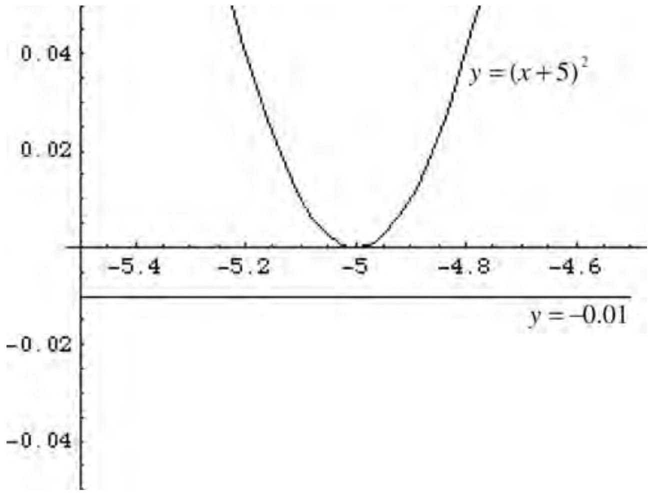


The graphs show that $(x-3)^2 \ge 0.01$ only when x is between 2.9 and 3.1. Since the statement includes equality, both 2.9 and 3.1 are part of the solution set.

(e) First, solve the equality

$$(x+5)^2 = -0.01$$
$$\sqrt{(x+5)^2} = \sqrt{-0.01}$$

Oops! There is no real number that you can square to get -0.01, so you cannot solve the equation. Graph the equations $y = (x+5)^2$ and $y = -0.01$ on your graphing calculator, with the window ranges $-5.5 \le x \le -4.5$ and $-0.05 \le y \le 0.05$:

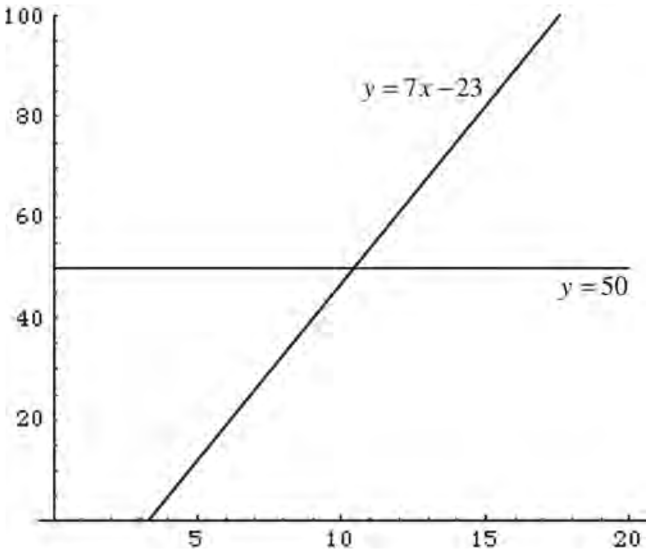


Since the graph of $y = (x+5)^2$ is always above the graph of $y = -0.01$, and the two graphs never intersect, the inequality is never true.

(f) Solve the equality

$$\begin{aligned} 7x - 23 &= 50 \\ 7x &= 73 \\ x &= \frac{73}{7} = 10\frac{3}{7} \end{aligned}$$

Use your graphing calculator to graph the two lines $y = 7x - 23$ and $y = 50$ with window ranges $0 \leq x \leq 20$ and $0 \leq y \leq 100$:



The graphs show that $7x - 23 > 50$ only when x is larger than $10\frac{3}{7}$.

(g) Solve the equality

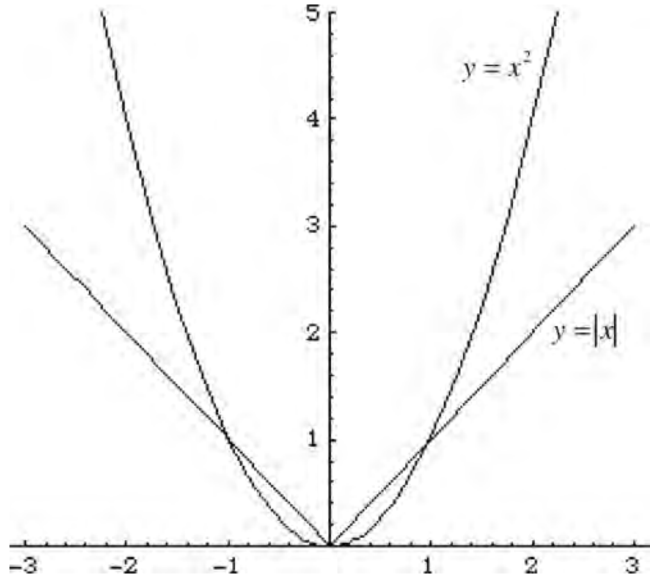
$$\begin{aligned} x^2 &= |x| \\ x^2 &= \pm x \end{aligned}$$

The next step may be a little tricky. You could divide both sides by x if you were sure that $x \neq 0$. So consider the two cases $x = 0$ and $x \neq 0$. If $x = 0$, the statement is true, so 0 is one solution. If $x \neq 0$, you can divide both sides by x.

$$\begin{aligned} x^2 &= \pm x \\ \frac{x^2}{x} &= \frac{\pm x}{x} \\ x &= \pm 1 \end{aligned}$$

So x can also equal 1 or -1 (if you are not sure, plug 0, 1, and -1 each into the original equation to make sure you have a true statement).

Next graph the two equations $y = x^2$ and $y = |x|$ on your graphing calculator using the window ranges $-5 \leq x \leq 5$, $0 \leq y \leq 10$:



The graphs show that $x^2 = |x|$ when x is -1, 0, or 1, and that $x^2 \geq |x|$ when x is less than or equal to -1, when $x = 0$, and when x is greater than 1.

3. (a) Solve the equality

$$\begin{aligned} -27.4x + 13 &= -27.2x + 12 \\ -27.4x + 13 + 27.4x &= -27.2x + 12 + 27.4x \\ 13 &= 0.2x + 12 \\ 1 &= 0.2x \\ 5 &= x \end{aligned}$$

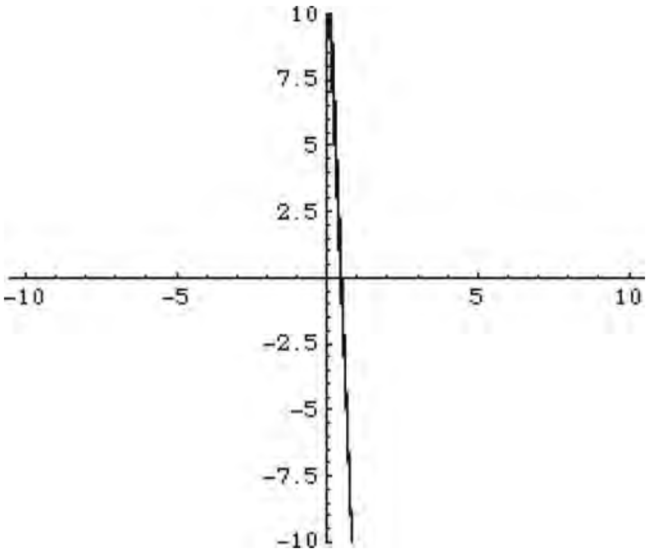
To find solutions to the inequality, test a convenient point on either side of $x = 5$, like $x = 0$. Considering that the equation has decimals, $x = 10$ is a good test point on the other side:

$$\begin{aligned} -27.4x + 13 &> -27.2x + 12 \\ -27.4(0) + 13 &\overset{?}{>} -27.2(0) + 12 \\ 13 &> 12 \quad \checkmark \end{aligned}$$

$$\begin{aligned} -27.4(10) + 13 &\overset{?}{>} -27.2(10) + 12 \\ -261 &\not> -260 \end{aligned}$$

So any number less than 5 (including zero) is a solution to the inequality.

(b) The figure below shows the two graphs in the standard viewing window, $-10 \le x \le 10$, $-10 \le y \le 10$:



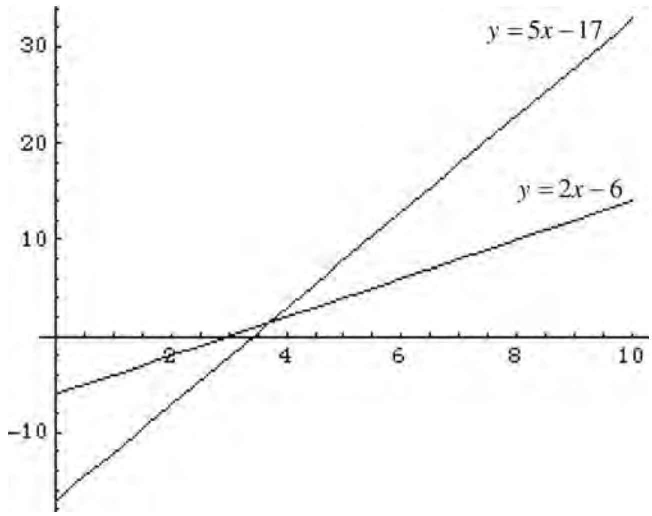
This image does not give any useful information about the intersection of the lines since the two lines are so close together. You could adjust the graphing calculator's window to find the intersection, but it is definitely easier here to solve the inequality by algebra.

4. (a) Solve the equality

$$\begin{aligned} 5x - 17 &= 2x - 6 \\ 5x - 17 - 2x &= 2x - 6 - 2x \\ 3x - 17 &= -6 \\ 3x - 17 + 17 &= -6 + 17 \\ 3x &= 11 \\ x &= \frac{11}{3} \end{aligned}$$

Testing values on either side of the equality (such as $x = 0$ and $x = 5$, for example) shows that $x > \frac{11}{3}$ is the correct solution.

Another way to solve the inequality is to graph $y = 5x - 17$ and $y = 2x - 6$ on the same axes, then find their intersection.



The graphs show that $y = 5x - 17$ is higher when x is larger than $3\frac{2}{3}$.

(b) In part (a), you found the set of numbers where $5x - 17 > 2x - 6$ and also where $5x - 17 = 2x - 6$. So, the *rest* of the numbers must be where $5x - 17 < 2x - 6$, that is, when $x < \frac{11}{3}$.

(c) You can solve the equality as usual:

$$\begin{aligned} 5(x-3) - 17 &= 2(x-3) - 6 \\ 5x - 15 - 17 &= 2x - 6 - 6 \\ 5x - 32 &= 2x - 12 \\ 5x - 32 - 2x &= 2x - 12 - 2x \\ 3x - 32 &= -12 \\ 3x &= 20 \\ x &= \frac{20}{3} \end{aligned}$$

You can also note that the difference between this part and part (a) is that x is replaced with $(x - 3)$. So you could keep the $(x - 3)$ together as long as possible, instead:

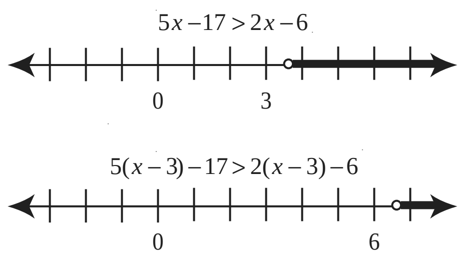
$$\begin{aligned} 5(x-3) - 17 &= 2(x-3) - 6 \\ 5(x-3) - 17 - 2(x-3) &= 2(x-3) - 6 - 2(x-3) \\ 3(x-3) - 17 &= -6 \\ 3(x-3) &= 11 \\ (x-3) &= \frac{11}{3} \end{aligned}$$

Notice that this process follows the identical steps as part (a). The key idea to realize is that you can just add 3 to the final answer you got in part (a).

$$\begin{aligned} x - 3 &= \frac{11}{3} \\ x - 3 + 3 &= \frac{11}{3} + 3 \\ x &= \frac{11}{3} + \frac{9}{3} = \frac{20}{3} \end{aligned}$$

So the solution to the inequality is $x > \frac{20}{3}$.

(d) The figure below shows the number lines for the two inequalities:



The solution to part (c) is 3 units to the right of the solution to part (a).

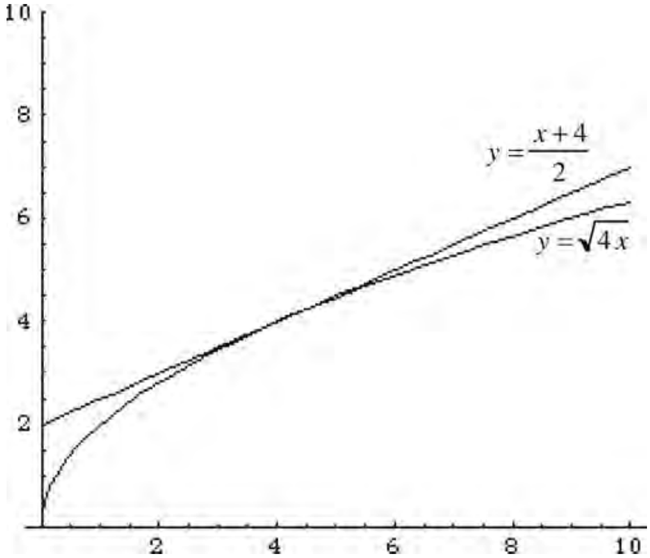
(e) The comparison of parts (a) and (c) showed that the solution to part (c) was the same as that of part (a), only shifted 3 units to the right. So the solution to the inequality when you replace x with $(x - 11)$ should be 11 units to the right, compared to the solution when using x. You can check the answer by testing a number close to the solution on each side. Here is the test for 14 (which should fail) and for 15 (which should pass):

$$5(x-11)-17>2(x-11)-6$$
$$5(14-11)-17\stackrel{?}{>}2(14-11)-6$$
$$5(3)-17\stackrel{?}{>}2(3)-6$$
$$-2\not>0$$

$$5(15-11)-17\stackrel{?}{>}2(15-11)-6$$
$$5(4)-17\stackrel{?}{>}2(4)-6$$
$$3>2\quad\checkmark$$

(f) Solve this inequality like the others. First, note that $x+8=x-(-8)$. In other words, the solution is moved to the *left* by 8 units to $\frac{11}{3}-8=-\frac{13}{3}$.

5. The figure below shows the graphs of the two equations for window ranges $0\le x\le 10$ and $0\le y\le 10$:



The graphs show that $\frac{x+4}{2}$ is larger than $\sqrt{4x}$ in most places, and that the graphs seem to be very close near $x=4$. Testing $x=4$ in each equation shows the point is an intersection. A table of values, a very accurate graph, or testing values on either end of $x=4$ can be used to show that the line is higher than the square root graph for all values other than 4. For example, here is a table for values of x between 3.5 and 4.5:

x	$\frac{x+4}{2}$	$\sqrt{4x}$
3.5	3.75	3.7417
3.6	3.8	3.7947
3.7	3.85	3.8471
3.8	3.9	3.8987
3.9	3.95	3.9497
4	4	4
4.1	4.05	4.0497
4.2	4.1	4.0988
4.3	4.15	4.1473
4.4	4.2	4.1952
4.5	4.25	4.2426

While this table is not a proof, it is convincing evidence that suggests that $\frac{x+4}{2}$ is always larger than $\sqrt{4x}$ for any numbers other than 4.

6. (a) When you have a fraction with the variable in the denominator, you need to take care that the denominator does not equal 0. So one possible cutoff point will be found where the equation is not defined. In this case, $x+3$ cannot equal 0, so x cannot equal -3 in the equation or the inequality.

(b) To solve the equation, you first need to multiply both sides by $x+3$. The only danger in multiplying both sides by a variable is that you could be multiplying by 0, which is not a valid move. But you've already noted that $x\ne -3$, so this move is okay.

$$\frac{x-5}{x+3}\cdot(x+3)=2\cdot(x+3)$$
$$x-5=2(x+3)$$
$$x-5=2x+6$$
$$-x=11$$
$$x=-11$$

Substitute this value into the original equation to make sure it works.

$$\frac{x-5}{x+3}=2$$
$$\frac{(-11)-5}{(-11)+3}\stackrel{?}{=}2$$
$$\frac{-16}{-8}\stackrel{?}{=}2$$
$$2=2\quad\checkmark$$

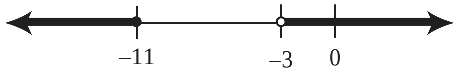
(c) Since the inequality uses the sign $\le$, the solution to the equality, $x=-11$, is included, and is indicated by a closed circle. The value $x=-3$ makes the inequality undefined, so it is indicated by an open circle.

(d) Test a value in each chunk. Some easy choices are $x=-20$, $x=-10$, and $x=0$. Here are the results of the testing:

$$\frac{x-5}{x+3}\le 2 \qquad\qquad \frac{x-5}{x+3}\le 2$$
$$\frac{(-20)-5}{(-20)+3}\stackrel{?}{\le}2 \qquad\qquad \frac{(-10)-5}{(-10)+3}\stackrel{?}{\le}2$$
$$\frac{-25}{-17}\stackrel{?}{\le}2 \qquad\qquad \frac{-15}{-7}\stackrel{?}{\le}2$$
$$1\frac{8}{17}\le 2\quad\checkmark \qquad\qquad 2\frac{1}{7}\not\le 2$$

$$\frac{x-5}{x+3} \leq 2$$
$$\frac{(0)-5}{(0)+3} \stackrel{?}{\leq} 2$$
$$\frac{-5}{3} \stackrel{?}{\leq} 2$$
$$-1\frac{2}{3} \leq 2 \quad \checkmark$$

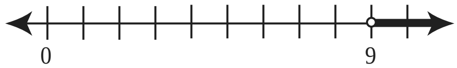
Shading in the intervals $x < -11$ and $x > -3$ gives this number line solution:



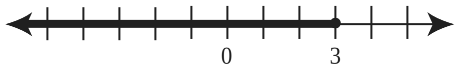
Note the closed circle at $x = -11$ indicates that value is included as a solution, while the open circle at $x = -3$ indicates that value is not included.

On Your Own

7. (a) First, solve the equation $2x = 18$ to get $x = 9$. So 9 is a cutoff point. Test a convenient point from one of the two regions, such as 0: $2(0) > 18$ is not true, so the region with 0 is not shaded. Try a number from the other region, such as 10: $2(10) > 18$ is true. So, to summarize, any number x such that $x > 9$ will work. Use an open circle for 9, since 9 is not part of the solution set.



(b) Just as before, solve the equation $9x = 27$ to get $x = 3$, so 3 is a cutoff point. Again, 0 is a convenient number to check the region less than 3: $9(0) \leq 27$ is true, so the region including 0 is part of the solution set. Try 5 to test the region greater than 3: $9(5) \leq 27$ is not true, so that region is not part of the solution set. So any number x such that $x \leq 3$ will work. Use a closed circle for 3, since 3 is part of the solution set.



(c) Solve the equation $-4x + 11 = 43$.

$$-4x + 11 = 43$$
$$-4x = 43 - 11$$
$$-4x = 32$$
$$x = -8$$

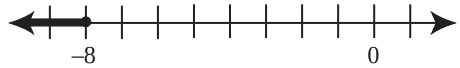
So -8 is a cutoff point. Again, 0 is a convenient number to check the region greater than -8:

$$-4x + 11 \geq 43$$
$$-4(0) + 11 \stackrel{?}{\geq} 43$$
$$11 \not\geq 43 \quad \textbf{X}$$

So 0 is not part of the solution. Test a number from the region less than -8, such as -10.

$$-4x + 11 \geq 43$$
$$-4(-10) + 11 \stackrel{?}{\geq} 43$$
$$40 + 11 \stackrel{?}{\geq} 43$$
$$51 \geq 43 \quad \checkmark$$

So -10 is part of the solution. As a result, any number x such that $x \leq -8$ will work. Note that your answer includes $<$ and not $>$, which could be surprising. For that reason, it is important for you to test regions to make sure you are including the correct one(s).
Use a closed circle for -8, since -8 is part of the solution set.



(d) Solve the equation $17 - 9x = 2x - 16$.

$$17 - 9x = 2x - 16$$
$$17 + 16 = 2x + 9x$$
$$33 = 11x$$
$$3 = x$$

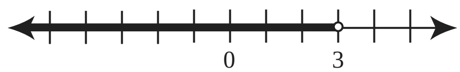
So 3 is a cutoff point. Use $x = 0$ to check the region $x < 3$:

$$17 - 9x > 2x - 16$$
$$17 - 9(0) \stackrel{?}{>} 2(0) - 16$$
$$17 > -16 \quad \checkmark$$

So 0 is part of the solution. Test a number from the other region, such as 10.

$$17 - 9(10) \stackrel{?}{>} 2(10) - 16$$
$$17 - 90 \stackrel{?}{>} 20 - 16$$
$$-73 \not> 7 \quad \textbf{X}$$

So 10 is not part of the solution. Thus, any x such that $x < 3$ will work. Use an open circle for 3, since 3 is part of the solution set.



(e) Solve the equation $2(3x + 1) = x + 6$.

$$2(3x + 1) = x + 6$$
$$6x + 2 = x + 6$$
$$5x + 2 = 6$$
$$5x = 4$$
$$x = \frac{4}{5}$$

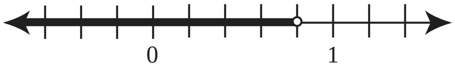
So $\frac{4}{5}$ is a cutoff point. Again, 0 is a convenient number to check:

$$2(3x+1) < x+6$$
$$2(3(0)+1) \overset{?}{<} (0)+6$$
$$2 < 6 \quad \checkmark$$

So 0 is part of the solution. Test a number from the other region, such as 10.

$$2(3(10)+1) \overset{?}{<} (10)+6$$
$$2(31) \overset{?}{<} 16$$
$$62 \not< 16$$

So 10 is not part of the solution. Thus, any x such that $x < \frac{4}{5}$ will work. Use an open circle for $\frac{4}{5}$, since $\frac{4}{5}$ is part of the solution set.



(f) Solve the equation $4x - 5 = 23$.

$$4x - 5 = 23$$
$$4x = 28$$
$$x = 7$$

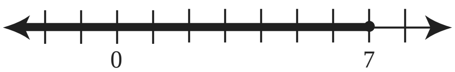
So 7 is a cutoff point. Again, 0 is a convenient number to check:

$$4x - 5 \le 23$$
$$4(0) - 5 \overset{?}{\le} 23$$
$$-5 \le 23 \quad \checkmark$$

So 0 is part of the solution. Test a number from the other region, such as 10.

$$4(10) - 5 \overset{?}{\le} 23$$
$$35 \not\le 23$$

So 10 is not part of the solution. Thus, any x such that $x \le 7$ will work. Use a closed circle for 7, since 7 is part of the solution set.



(g) Solve the equation $13 + x = 13 - x$.

$$13 + x = 13 - x$$
$$2x = 0$$
$$x = 0$$

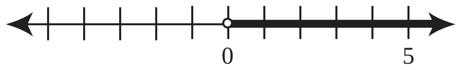
So 0 is a cutoff point. 0 is not a convenient number to check this time, since it is the cutoff point! So try one on each side. First, try 5:

$$13 + x > 13 - x$$
$$13 + (5) \overset{?}{>} 13 - (5)$$
$$18 > 8 \quad \checkmark$$

So 5 is part of the solution. Test a number from the other region, such as -5.

$$13 + x > 13 - x$$
$$13 + (-5) \overset{?}{>} 13 - (-5)$$
$$8 \not> 18$$

So -5 is not part of the solution. Thus, any x such that $x > 0$ will work. Use an open circle for 0, since 0 is part of the solution set.



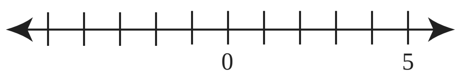
(h) Solve the equation $-2(3 - 2x) = 4x + 7$.

$$-2(3 - 2x) = 4x + 7$$
$$-6 + 4x = 4x + 7$$
$$-6 = 7$$

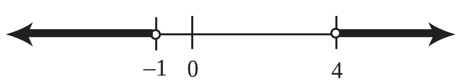
That last statement is false. So there are no solutions to the equation. What can you do now? Well, since there is no cutoff point, there is only one region to test. So test the number 0.

$$-2(3 - 2x) \ge 4x + 7$$
$$-2(3 - 2(0)) \overset{?}{\ge} 4(0) + 7$$
$$-6 \not\ge 7$$

Since 0 is not part of the solution, no number will be a solution to the inequality. The number line would thus be empty.

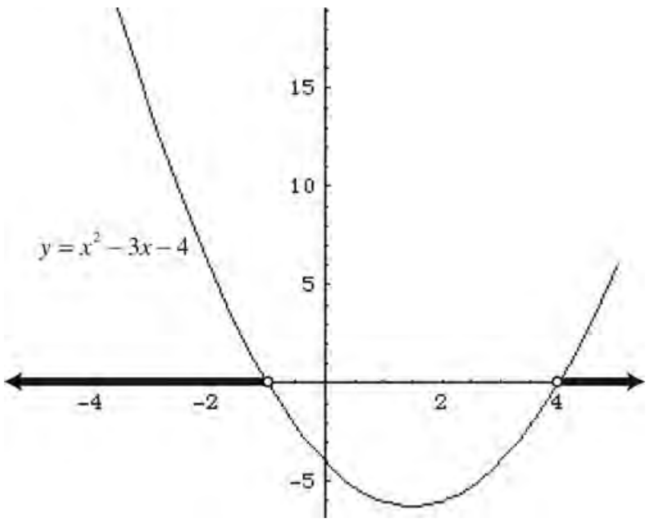


8. Since the graph is $y = x^2 - 3x - 4$, the solutions are the values of x that make $y > 0$. Wherever the graph is above the x-axis, that value of x is a solution. Since the graph crosses $y = 0$ at the values $x = -1$ and $x = 4$, the number-line solution is



The number line solution can even be drawn on the

x-axis of the graph.



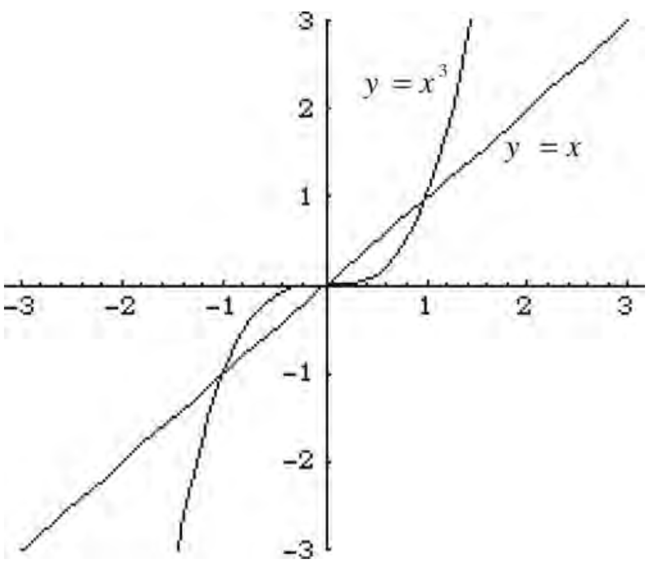
9. There are a number of ways you can determine what the solution set is.

- Make a table of values for x^3 and x to find places where one is larger than the other:

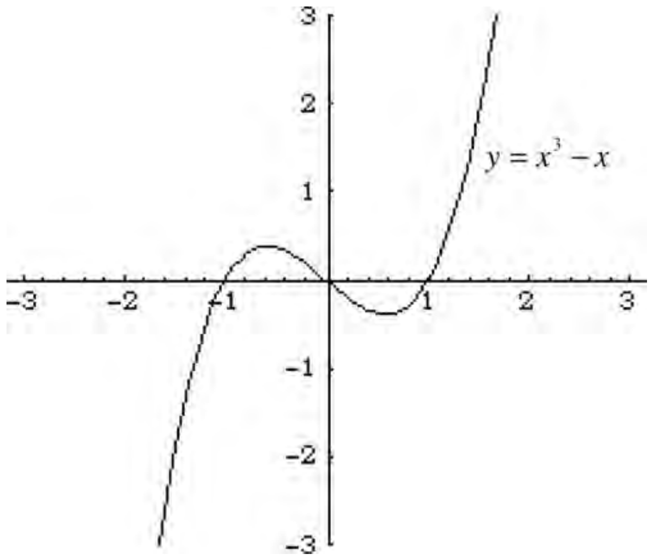
x	x^3
-2	-8
-1.5	-3.375
-1	-1
-0.5	-0.125
0	0
0.5	0.125
1	1
1.5	3.375
2	8

The table indicates that the two sides are equal when $x = -1$, 0, and 1, and shows the areas where x^3 is larger: between -1 and 0, and greater than 1.

- Draw two graphs on the same axes, $y = x^3$ and $y = x$. These graphs intersect in three places, and you can see where one is larger than the other:



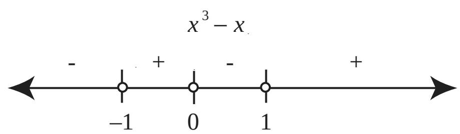
- Subtract x from each side, producing the inequality $x^3 - x > 0$. You could then graph $y = x^3 - x$ and find out where it is positive:



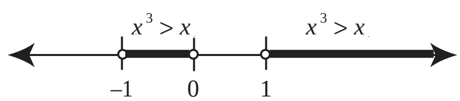
- Use algebra to find the three values of x where $x^3 = x$.

$$
\begin{aligned}
x^3 &= x \\
x^3 - x &= 0 \\
x(x^2 - 1) &= 0 \\
x(x+1)(x-1) &= 0 \\
x &= \{0, -1, 1\}
\end{aligned}
$$

Then, try a number within each interval to see whether $x^3 > x$ is true in the four zones broken up by these solutions:



Any way you go about it, the solution set is $-1 < x < 0$ and $x > 1$.



10. (a) Replace x with 2.5 to test the value:

$$
\begin{aligned}
x^3 + 28x &< 10x^2 + 24 \\
2.5^3 + 28(2.5) &\stackrel{?}{<} 10(2.5)^2 + 24 \\
15.625 + 70 &\stackrel{?}{<} 10(6.25) + 24 \\
85.625 &< 86.25 \quad \checkmark
\end{aligned}
$$

So 2.5 makes the equation true.

(b) Replace x with each value to test it:

$$x^3 + 28x < 10x^2 + 24$$
$$3^3 + 28(3) \stackrel{?}{<} 10(3)^2 + 24$$
$$111 < 114 \quad \checkmark$$

$$(3.5)^3 + 28(3.5) \stackrel{?}{<} 10(3.5)^2 + 24$$
$$140.875 < 146.5 \quad \checkmark$$

$$5^3 + 28(5) \stackrel{?}{<} 10(5)^2 + 24$$
$$265 < 274 \quad \checkmark$$

All three numbers make the inequality true.

(c) You already know where the two sides are equal. The only way for there to be a change in the inequality is at a cutoff point—an equality value or a value that makes x undefined (there are none of these for this inequality). So all the values between 2 and 6 must be on the same side of the inequality.

(d) Choose numbers in each interval that are easy to work with. Some good choices are 0 and 10 within the intervals you don't know yet (the interval from 2 to 6 is known). Try 0 and 10 to determine which intervals work:

$$x^3 + 28x < 10x^2 + 24$$
$$0^3 + 28(0) \stackrel{?}{<} 10(0)^2 + 24$$
$$0 < 24 \quad \checkmark$$

$$10^3 + 28(10) \stackrel{?}{<} 10(10)^2 + 24$$
$$1280 \not< 1024$$

So, the interval when $x < 2$ and when $2 < x < 6$ are solutions, while the interval when $x > 6$ is not. Note that the solution does *not* include the value $x = 2$.

11. (a) The number line below shows $x > 3$. Note an open circle at $x = 3$.

(b) The number line below shows $x \le 10$. Note a closed circle at $x = 10$.

(c) Combine these number lines by including only numbers that are part of both number lines.

(d) Any number is either larger than 3 or less than or equal to 10. So, the number line solution would just be the entire line.

12. (a) Replace x with the number 7:

$$\frac{6}{x-4} = 2$$
$$\frac{6}{7-4} \stackrel{?}{=} 2$$
$$\frac{6}{3} = 2 \quad \checkmark$$

So the equation is true when $x = 7$.

(b) x cannot be 4, since the denominator would then equal 0. Any other number can be used.

(c) Solve for x by multiplying each side by $(x - 4)$ (which is valid as long as $x \neq 4$), then solving using the basic rules and moves.

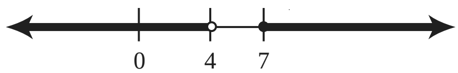
$$\frac{6}{x-4} = 2$$
$$\frac{6}{x-4} \cdot (x-4) = 2 \cdot (x-4)$$
$$6 = 2(x-4)$$
$$6 = 2x - 8$$
$$6 + 8 = 2x - 8 + 8$$
$$14 = 2x$$
$$7 = x$$

(d) The equation is false for all numbers x besides 4 and 7. 7 makes it true, while 4 cannot be used at all.

(e) There are many possible answers. Five are 8, 10, 3, -2, and 0.

(f) There are many possible answers. Five are 5, 6, 4.1, $6\frac{2}{3}$, and 2π.

(g) The number line includes 7 as a solution, but not 4.



13. (a) The first inequality is $2x - 4 \ge 3x + 11$. Plug in 0 to get $2(0) - 4 \ge 3(0) + 11$, which simplifies to $-4 \ge 11$, which is false.

The second inequality is $2x \ge 3x + 15$. Plug in 0 to get $2(0) \ge 3(0) + 15$, which simplifies to $0 \ge 15$, which also is false.

The third inequality is $-x \ge 15$. Plug in 0 to get $-(0) \ge 15$, or simply $0 \ge 15$, which also is false.

The last inequality is $x \ge -15$. Plug in 0 to get $0 \ge -15$, which is true!

(b) Dividing each side by -1 changed the value of the inequality from false to true. So somehow that step is wrong.

14. (a) This statement is always true.

Think about it this way: suppose you have two numbers, a and b. If $a < b$, then the difference between b and a, that is, $b - a$, must be positive. Likewise, if you know $b - a$ is positive, then it must be true that $a < b$.

So now, you want to check to see if $a + c < b + c$. Well, if it is, then $(b + c) - (a + c)$ must be positive. Distribute the minus sign to get $b + c - a - c$. The two c's cancel each other out, and you're left with $b - a$. You already know $b - a$ is positive, so then $b + c - a - c$ is positive, and $(b + c) - (a + c)$ is positive, so then $a + c < b + c$.

For example, 5 is smaller than 8. (5 is a, and 8 is b). And $8 - 5 = 3$, which is positive. Now, is $5 + 6 < 8 + 6$? Yes, since $11 < 14$, but also because $(8+6)-(5+6) = 8+6-5-6 = 8-5+6-6 = 8-5 = 3$, which is positive.

(b) This statement is sometimes true. For example, let $a = 3$, $b = 5$, and $c = 2$. It is true that $a < b$, since $3 < 5$. And $ac < bc$, since $3 \cdot 2 = 6 < 10 = 5 \cdot 2$.

But it is sometimes false. For example, let $a = -2$, $b = 4$, and $c = -3$. It is true that $a < b$, since $-2 < 4$. But $ac \nless bc$, since $(-2) \cdot (-3) = 6 \nless -12 = 4 \cdot (-3)$.

Negative numbers seem to mess up this conjecture. If a, b and c are all positive, then the statement is always true. But you do not have to be so restrictive. In fact, as long as c is positive, it doesn't matter what sign a and b are.

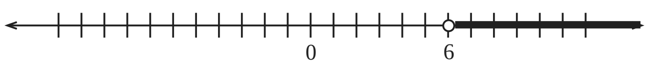
Can the conjecture be proven? Well, follow the same logic as in part (a). If $a < b$, then $b - a$ must be positive. So now look at $ac < bc$. If that's true, then $bc - ac$ must be positive. You can use the distributive property in reverse and say that $c(b - a)$ must be positive. Well, you already know that $b - a$ is positive. So that *product* $c(b - a)$ is positive only if c is also positive.

If c is negative, then $c(b - a)$ is negative, which means $bc - ac$ is negative. So then $ac - bc$ is positive. And that means that $bc < ac$, or $ac > bc$. Aha! So if $c > 0$, $a < b \iff ac < bc$. And if $c < 0$, $a < b \iff ac > bc$.

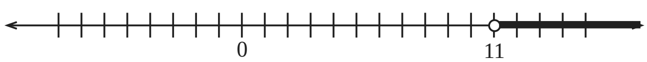
15. The correct answer is **D**. To see this, start with the inequality $6 \geq 2 - n > -2$. Subtract 2 to obtain $4 \geq -n > -4$. Finally, divide by -1 to get $-4 \leq n < 4$.

Maintain Your Skills

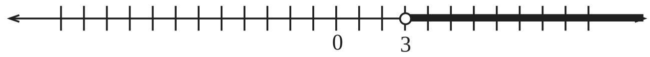
16. (a) $x > 6$



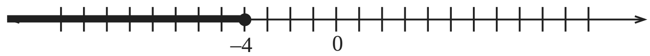
(b) $(x - 5) > 6$



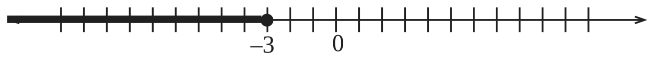
(c) $(x + 3) > 6$



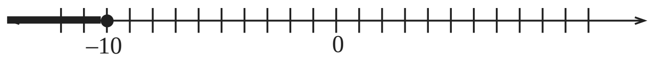
(d) $2x + 12 \leq 4$



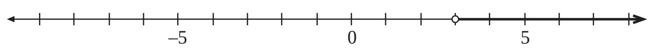
(e) $2(x - 1) + 12 \leq 4$



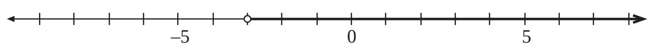
(f) $2(x + 6) + 12 \leq 4$



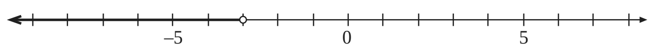
17. (a)



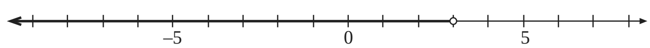
(b)



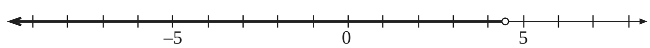
(c)



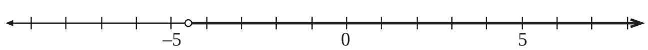
(d)



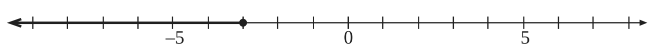
(e)



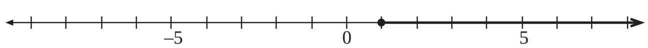
(f)



(g)



(h)



4.15 Linear Trends in Data

Check Your Understanding

1. (a) The variable x is the temperature, and the variable y is the number of attendees, in thousands.

(b) Show that the balance point is on the line by replacing x with 80 and y with 47.1 and then checking to see if the equation is (approximately) true:

$$y = 0.678x - 7.1$$
$$47.1 \stackrel{?}{=} 0.678(80) - 7.1$$
$$47.1 \stackrel{?}{=} 54.24 - 7.1$$
$$47.1 \approx 47.14 \quad \checkmark$$

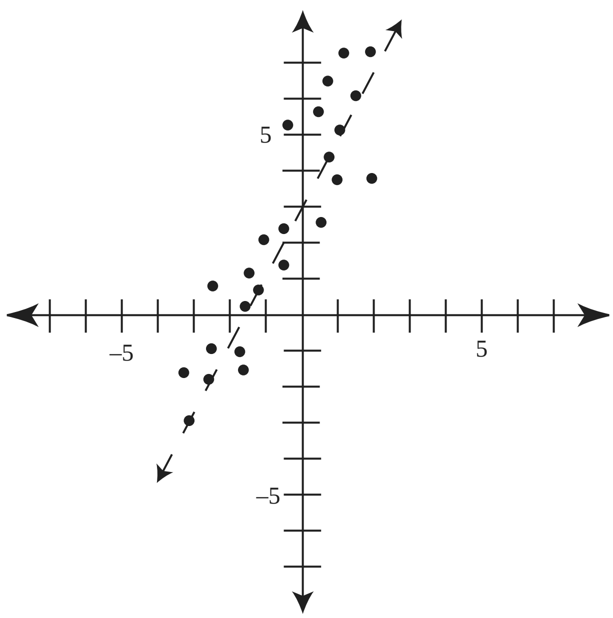
While these answers are not exactly equal, they are extremely close, and more precision could be attained by using greater accuracy in the line's equation.

2. (a) The balance point is the center of the ruler (at 6 inches for a 12-inch ruler, for example, and halfway from top to bottom).

(b) The balance point is the center of the circular coin.

(c) It is possible to have a balance point that is not within the object. One example is a boomerang. Another is a cup or mug: its balance point will be in the center, but not part of the actual object.

3. The correct answer is **A**, $y = 2x + 3$.
Only the line $y = 2x + 3$ follows the data.

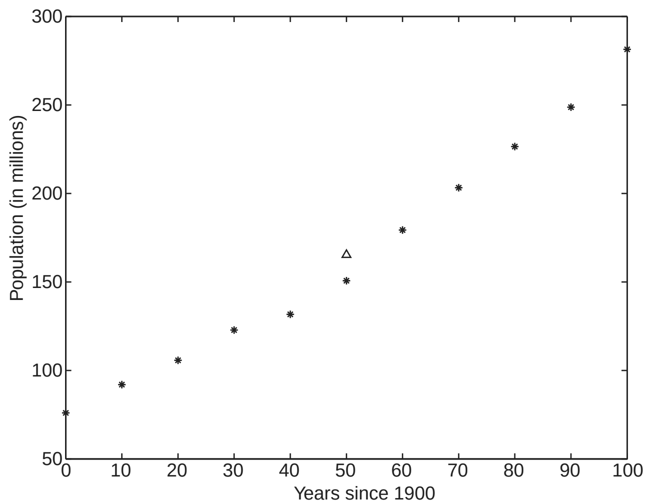


The other lines do not follow the trend. $y = 2x - 3$ is below the trend, while the others do not follow it at reasonable slopes.

4. (a) Answers will vary, but one way is to find the difference between consecutive outputs. These differences are not the same, so the points are not on a line.

(b) Answers may vary. The plot of the data points below shows the population growing, but not along a line. It could be roughly approximated by a line. A better approximation might be two line segments.

(c) The balance point is about $(1950, 165.27)$. This point is above the trend in the data, which suggests there is not a linear trend. Here is a graph of the 11 data points and the balance point:



On Your Own

5. Tables 1, 2, and 4 show linear trends. Table 3 does not. See the solution to Exercise 6 for the plots.

6. Here are the four graphs. In each, the data points are marked as stars, while the balance point is marked as a triangle.

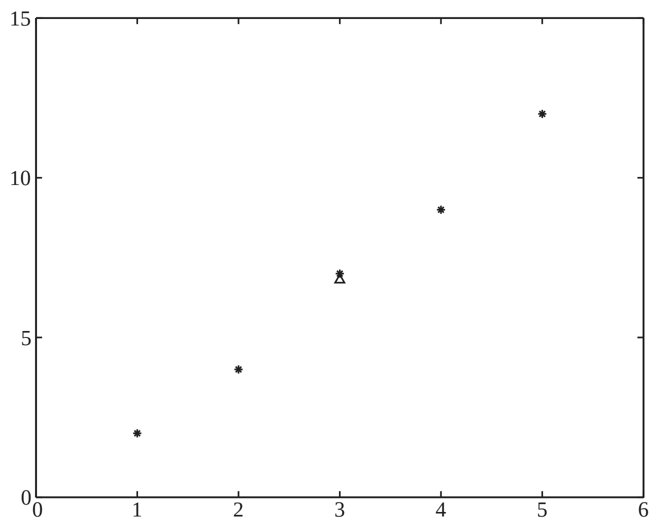


Table 1: Balance point $(3, 6.8)$

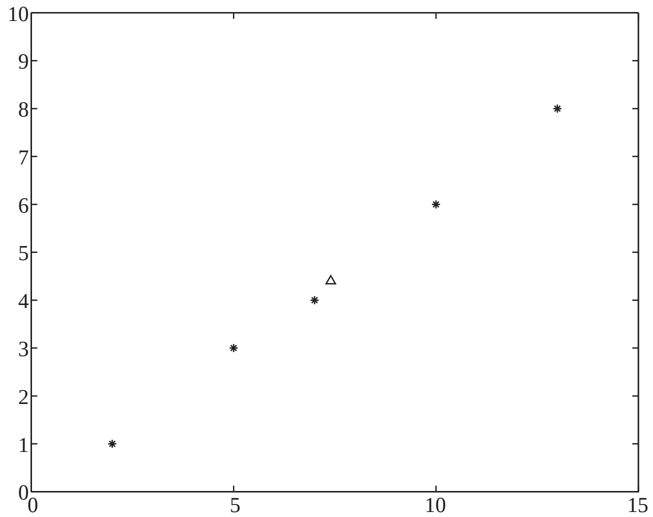


Table 2: Balance point $(7.4, 4.4)$

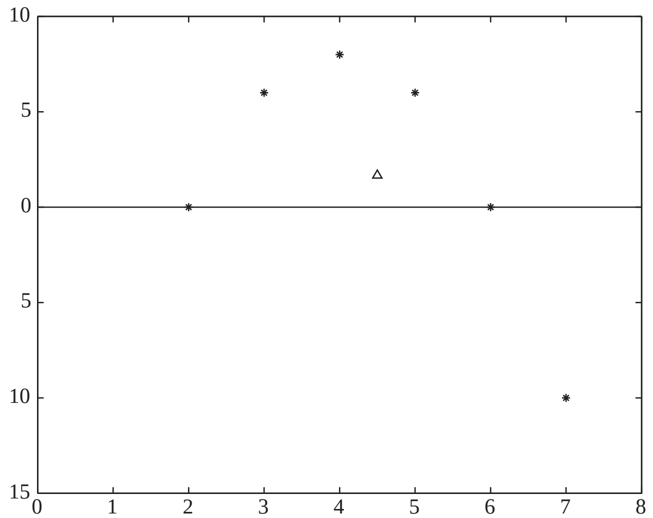


Table 3: Balance point $\left(4.5, 1\frac{2}{3}\right)$

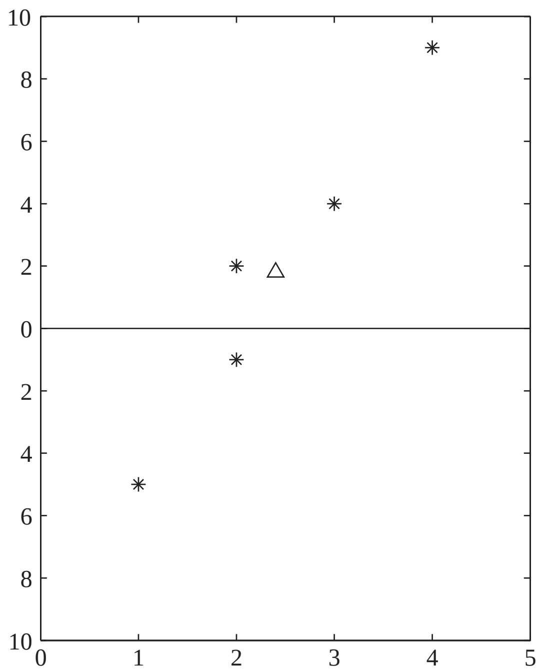


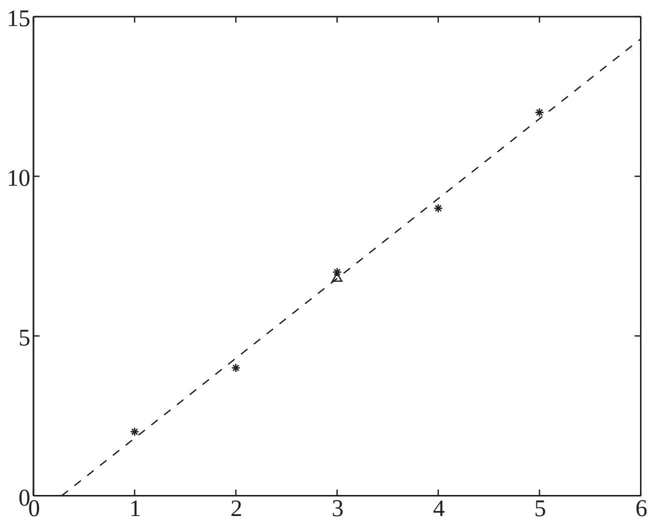
Table 4: Balance point $(2.4, 1.8)$

In all but Table 3, the balance point lies along the trend in data. Table 3 is the only table that cannot be fit well with a line. The data for Table 3 comes from a parabola, which does not have a linear equation.

7. For Table 1, a reasonable guess at the slope of the trend is anywhere between 2 and 3. A line connecting the first and last data points has slope 2.5 (rise of 10 divided by run of 4), so 2.5 is a very good guess. Using this estimate for the trend line's slope, the equation is

$$y - 6.8 = 2.5(x - 3)$$

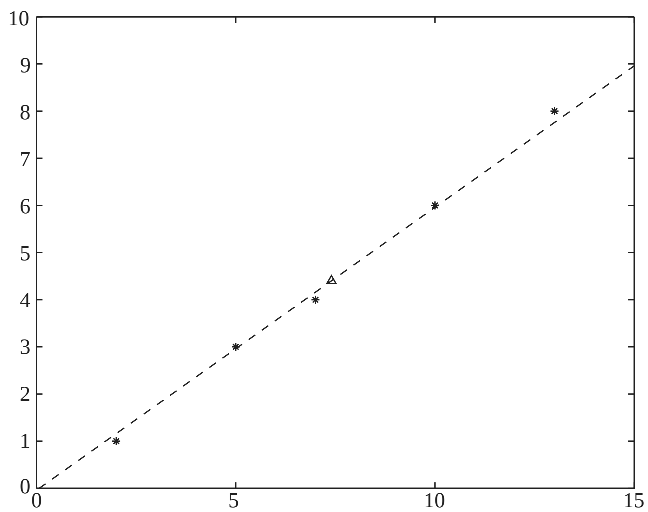
which simplifies to $y = 2.5x - 0.7$. It turns out that this is the exact line returned as best fit from a calculator. Here is the graph:



For Table 2, a reasonable guess at the slope of the trend is between $\frac{1}{2}$ and $\frac{2}{3}$. The slope between the endpoints is $\frac{7}{11} \approx 0.64$ (rise of 7 divided by run of 11). If slope 0.6 is used, the equation through the balance point is

$$y - 4.4 = 0.6(x - 7.4)$$

which simplifies to $y = 0.6x - 0.04$. Here is the graph:



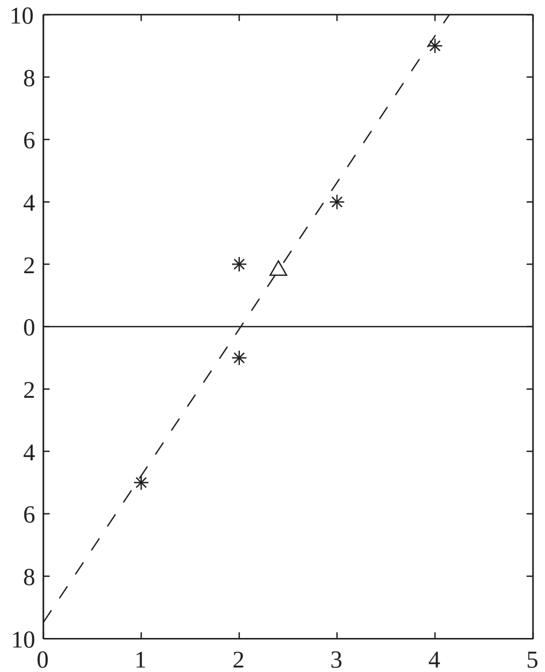
The actual best fit slope is $\frac{77}{122} \approx 0.6312$, and one equation for the best fit line is

$$y - 4.4 = \frac{77}{122}(x - 7.4)$$

For Table 4, a reasonable guess at the slope of the trend is between 4 and 5. The slope between the endpoints is $4\frac{2}{3}$ (rise of 14 divided by run of 3), so anything between 4.5 and 5 is an excellent guess. Using the estimate 4.7 for the trend line's slope, the equation is

$$y - 1.8 = 4.7(x - 2.4)$$

which simplifies to $y = 4.7x - 9.48$. Here is the graph:



The actual best fit slope is 4.5, and the best fit line is $y = 4.5x - 9$.

8. Answers will vary, but only (c) $y = 0.9x + 0.9$ and (d) $y = x + 0.5$ follow the trend well. Line (a) $y = x + 1$ is above most data points, and line (d) $y = 0.5x + 2.5$ is not steep enough to follow the trend.

9. (a) The predicted values are too high. This line is probably a poor fit, since almost all the errors are on one side.

(b) Here are the tables for the other lines:

Data vs. Line Fit for Line (b)

Input	Actual	Predicted: $y = 0.5x + 2.5$	Error: Actual − Predicted
1	1.8	3	−1.2
2	1.7	3.5	−1.8
3	3.6	4	−0.4
5	5.4	5	0.4
6	7.3	5.5	1.8
7	7.2	6	1.2

Data vs. Line Fit for Line (c)

Input	Actual	Predicted: $y = 0.9x + 0.9$	Error: Actual − Predicted
1	1.8	1.8	0
2	1.7	2.7	−1
3	3.6	3.6	0
5	5.4	5.4	0
6	7.3	6.3	1
7	7.2	7.2	0

Data vs. Line Fit for Line (d)

Input	Actual	Predicted: $y = x + 0.5$	Error: Actual − Predicted
1	1.8	1.5	0.3
2	1.7	2.5	−0.8
3	3.6	3.5	0.1
5	5.4	5.5	−0.1
6	7.3	6.5	0.8
7	7.2	7.5	−0.3

Line (c) makes only two errors; the other four points are exactly right. Line (d) has the smallest maximum error of ± 0.8 but does not match any data exactly. Depending on the criteria, either can be the best fit of the four lines. Both lines (c) and (d) pass through the balance point $(4, 4.5)$.

10. The line of best fit passes roughly through $(2010, 10{,}900)$. The correct answer is **C**.

Maintain Your Skills

11. (a) (i)

Data vs. Line Fit: $y - 4.5 = x - 4$

Input	Actual	Predicted	Error
1	1.8	−1.5	3.3
2	1.7	0.5	1.2
3	3.6	2.5	1.1
5	5.4	6.5	−1.1
6	7.3	8.5	−1.2
7	7.2	10.5	−3.3

(ii)

Data vs. Line Fit: $y - 4.5 = 2(x - 4)$

Input	Actual	Predicted	Error
1	1.8	−4.5	6.3
2	1.7	−1.5	3.2
3	3.6	1.5	2.1
5	5.4	7.5	−2.1
6	7.3	10.5	−3.2
7	7.2	13.5	−6.3

(iii)

Data vs. Line Fit: $y - 4.5 = 3(x - 4)$

Input	Actual	Predicted	Error
1	1.8	−4.5	6.3
2	1.7	−1.5	3.2
3	3.6	1.5	2.1
5	5.4	7.5	−2.1
6	7.3	10.5	−3.2
7	7.2	13.5	−6.3

(b) The sum of the errors in each table is 0.

(c)

Data vs. Line Fit: $y - 4.5 = \frac{1}{2}(x - 4)$

Input	Actual	Predicted	Error
1	1.8	3	−1.2
2	1.7	3.5	−1.8
3	3.6	4	−0.4
5	5.4	5	0.4
6	7.3	5.5	1.8
7	7.2	6	1.2

Data vs. Line Fit: $y - 4.5 = -18(x - 4)$

Input	Actual	Predicted	Error
1	1.8	58.5	−56.7
2	1.7	40.5	−38.8
3	3.6	22.5	−18.9
5	5.4	−13.5	18.9
6	7.3	−31.5	38.8
7	7.2	−49.5	56.7

Again, for both $m - \frac{1}{2}$ and $m = -18$, the sum of the errors are 0. Note especially that the line with slope -18 is a poor fit—it doesn't follow the trend at all! Yet the sum of the errors is still 0 (a good indication that the sum of the errors is not the right way to judge the quality of the fit of a line).

A good conjecture would be that the sum of the errors for any line through the balance point (with any slope) will be 0. Why would that be? Well, you can prove the conjecture using algebra.

The table below uses the same process as the tables you have already worked with. The difference is, you have expressions instead of numbers in the predicted and error columns.

Data vs. Line Fit: $y - 4.5 = m(x - 4)$

Input	Actual	Predicted	Error
1	1.8	$m(1-4)+4.5$	$1.8-(-3m+4.5)$
2	1.7	$m(2-4)+4.5$	$1.7-(-2m+4.5)$
3	3.6	$m(3-4)+4.5$	$3.6-(-m+4.5)$
5	5.4	$m(5-4)+4.5$	$5.4-(m+4.5)$
6	7.3	$m(6-4)+4.5$	$7.3-(2m+4.5)$
7	7.2	$m(7-4)+4.5$	$7.2-(3m+4.5)$

You can simplify each expression further, but notice (for now) that, for example, $m(1-4)+4.5 = -3m+4.5$, so the error column is just the actual column minus the predicted column. Now, add up the error column. You get

$$\begin{gathered}1.8-(-3m+4.5)+1.7-(-2m+4.5)+\\3.6-(-m+4.5)+5.4-(m+4.5)+\\7.3-(2m+4.5)+7.2-(3m+4.5)\end{gathered}$$

Rearrange the terms to get

$$\begin{gathered}1.8+1.7+3.6+5.4+7.3+7.2+\\3m+2m+m-m-2m-3m-6(4.5)\end{gathered}$$

You may have noticed that the first six terms is just the sum of the actual data. The next six terms (all of the m's) cancel out to 0, and that 4.5 is the y-coordinate of the balance point. In other words, 4.5 is the *average* of the actual data. So what are you left with? Well, you have

$$(\text{sum of the actual data}) - 6(\text{average of actual data})$$

Since there were six original data points,

$$\text{average of actual data} = \frac{\text{sum of actual data}}{6}$$

Substitute that back in to get

$$\begin{aligned}(\text{sum of the actual data}) - 6\left(\frac{\text{sum of actual data}}{6}\right)\\ &= (\text{sum of the actual data})\\ &\quad - (\text{sum of actual data})\\ &= 0\end{aligned}$$

So it does not matter what the slope is: if the line goes through the balance point, the sum of the errors will always be 0.

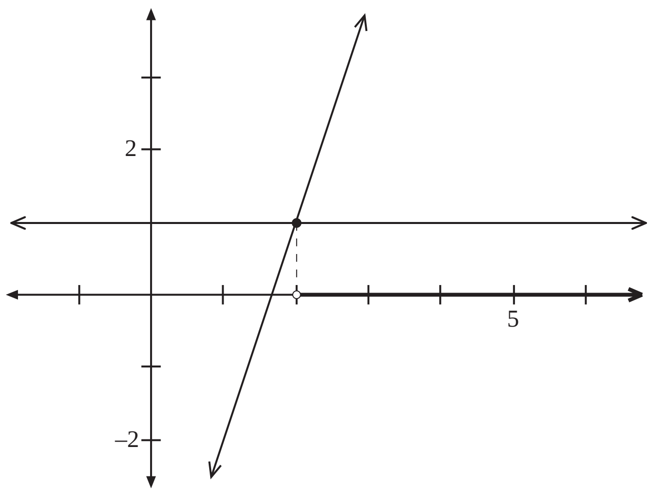
4D MATHEMATICAL REFLECTIONS

1. (a) The two lines intersect in one point. One solution.
(b) The parabola intersects the line in two points. Two solutions.
(c) The two lines are parallel. They do not intersect. No solutions.
(d) The parabola and the line do not intersect. No solutions.

2. (a) First, graph the two equations $y = 3x - 5$ and $y = 1$. To find the exact cutoff point, solve the equation $3x - 5 = 1$.

$$\begin{aligned}3x - 5 &= 1\\ 3x &= 6\\ x &= 2\end{aligned}$$

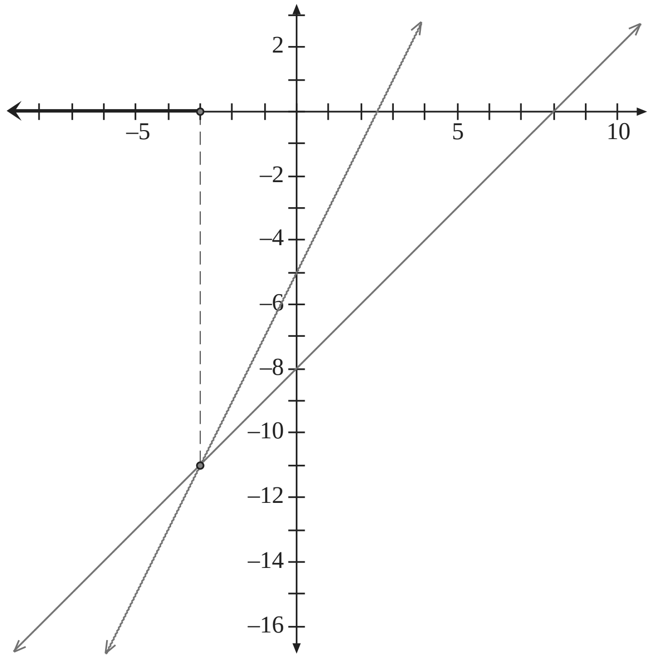
So, 2 is the cutoff point, and the y-height of the graph of $y = 3x - 5$ is higher than that of the graph of $y = 1$ to the right of the cutoff point.



(b) Solve $x - 8 = 2x - 5$ to find the cutoff point.

$$\begin{aligned}x - 8 &= 2x - 5\\ x - 2x - 8 &= 2x - 2x - 5\\ -x - 8 &= -5\\ -x &= 3\\ x &= -3\end{aligned}$$

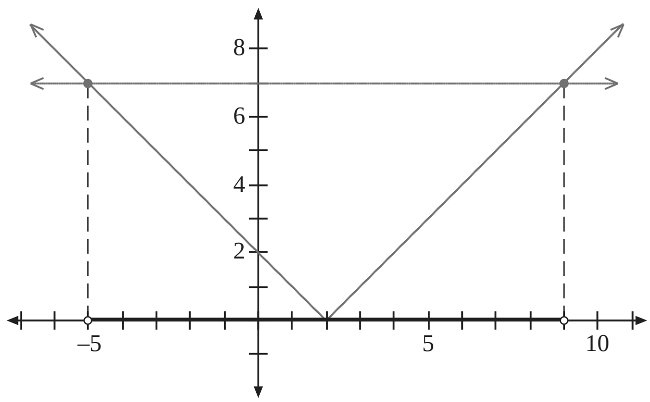
-3 is the cutoff point. Graph the two equations to see where the y-height of the graph of $y = x - 8$ is greater than that of $y = 2x - 5$. Use a closed circle for -3, since -3 is part of the solution set.



(c) Solve the equation $|x - 2| = 7$ for the cutoff points:

$$x - 2 = 7 \rightarrow x = 9 \quad \text{or} \quad x - 2 = -7 \rightarrow x = -5$$

So the solution will have two cutoff points, 9 and -5. Graph the two equations to see where the y-height of the graph of $y = |x - 2|$ is below that of $y = 7$. Use open circles at -5 and 9 since they are not part of the solution set.



3. (a) First solve the equation:

$$\begin{aligned} 4x + 2 &= 5x - 3 \\ -x + 2 &= -3 \\ -x &= -5 \\ x &= 5 \end{aligned}$$

Graph $y = 4x + 2$ and $y = 5x - 3$. The graph shows that $4x + 2 > 5x - 3$ when $x < 5$.

(b) Solve the equation:

$$\begin{aligned} 2x + 3 &= 48 \\ 2x &= 45 \\ x &= 22.5 \end{aligned}$$

Graph $y = 2x + 3$ and $y = 48$ (You may want to change your calculator window!). The graph shows that $2x + 3 \le 48$ when $x \le 22.5$.

(c) Solve the equation $|2x + 1| = 5$.

$$\begin{aligned} 2x + 1 &= 5 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

or

$$\begin{aligned} 2x + 1 &= -5 \\ 2x &= -6 \\ x &= -3 \end{aligned}$$

So, the cutoff points are -3 and 2. Graph the equations $y = |2x + 1|$ and $y = 5$. The graph show that $|2x + 1| < 5$ only when x is between -3 and 2. So, $-3 < x < 2$.

4. (a) Find the average of the x's:

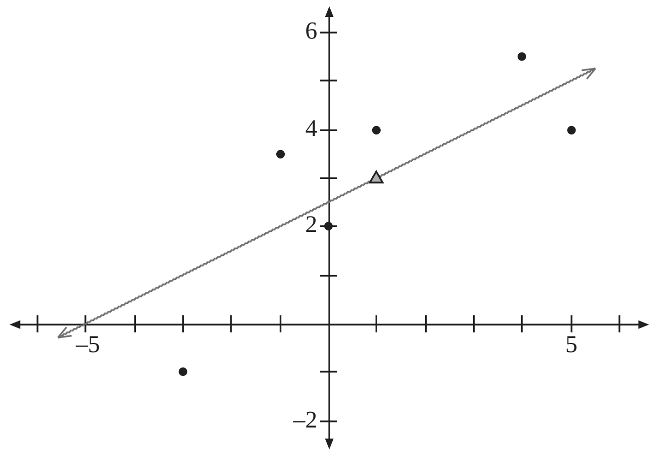
$$\frac{-1 + 1 + 4 + 5 + 0 + (-3)}{6} = \frac{6}{6} = 1$$

Find the average of the y's:

$$\frac{3.5 + 4 + 5.5 + 4 + 2 + (-1)}{6} = \frac{18}{6} = 3$$

The balance point is $(1, 3)$.

(b)



(c) Estimate the slope of the line. If your estimate is $\frac{1}{2}$, the point-tester equation will be

$$\begin{aligned} \frac{y - 3}{x - 1} &= \frac{1}{2} \\ y - 3 &= \frac{1}{2}(x - 1) \\ 2(y - 3) &= x - 1 \\ 2y - 6 &= x - 1 \\ -6 &= x - 1 - 2y \\ -5 &= x - 2y \end{aligned}$$

5. The solutions of the equations will break the number line into regions. Testing a point in each region will tell you which regions are part of the solution set.

6. A good fitting line will go through the balance point. Find the balance point by taking the mean of all the data points. That is

- The x-coordinate of the balance point will be the mean of the x-coordinates of the data points, $\bar{x}$.
- The y-coordinate of the balance point will be the mean of the y-coordinates of the data points, $\bar{y}$.

Once you find the balance point, make a guess at the slope that goes closest to all the points. You can check your line by comparing points from the data set with points on the line (comparing points with the same x-coordinate, find the difference between the y-coordinates).

7. The first taxi costs \$1.10 for the first mile and \$1 for each additional mile. Use guess-check-generalize to find an expression for the cost of the taxi ride:

- If the ride is 2 miles, then the cost is \$1.10 for the first mile, then \$1 for the second mile, for a total of \$2.10 for the ride.
- If the ride is 10 miles, then the cost is \$1.10 for the first mile, then \$1 for the rest of the trip, 10 miles − 1 miles = 9 miles. The total cost of the trip is \$1.10 + \$1 · 9 = \$1.1 + \$9 = \$10.10 for the whole trip.
- If the ride is m miles, then the cost is \$1.10 for the first mile, then \$1 for the rest of the trip, $m - 1$ miles. The total cost of the trip is $1.1 + 1 \cdot (m - 1)$.

Next, simplify the expression

$$1.1 + 1 \cdot (m - 1) = 1.1 + m - 1 = m + 0.1$$

Follow the same process for the second taxi. An expression for the cost of a ride in the second taxi is

$$1.7 + 0.6m$$

If you want to know when the first taxi is cheaper, you want to solve the inequality

$$m + 0.1 < 0.6m + 1.7$$

First, solve the corresponding equation to find when the two taxis cost the same:

$$\begin{aligned} m + 0.1 &= 0.6m + 1.7 \\ m - 0.6m &= 1.7 - 0.1 \\ 0.4m &= 1.6 \\ m &= 4 \end{aligned}$$

So the first taxi is cheaper for any trip less than 4 miles in length. Otherwise, the second taxi is cheaper.

CHAPTER REVIEW

1. (a) Answers will vary. First find $m(A, B)$:

$$m(A, B) = \frac{-2-1}{0-3} = \frac{-3}{-3} = 1$$

Then use the slope to find another point. The slope is $\frac{\text{change in } y}{\text{change in } x}$. Think of 1 as $\frac{1}{1}$ or $\frac{2}{2}$ or ... and use A as a base point:

$$C = (0 + 1, -2 + 1) = (1, -1)$$

or

$$C = (0 + (-1), -2 + (-1)) = (-1, -3)$$

(b) Answers will vary.

$$m(A, B) = \frac{2 - (-2)}{4 - 6} = \frac{4}{-2} = \frac{-2}{1}$$

$$C = (4 + (-1), 2 + 2) = (3, 4)$$

or

$$C = (4 + 1, 2 + (-2)) = (5, 0)$$

(c)

$$m(A, B) = \frac{3 - (-1)}{-8 - (-5)} = \frac{4}{-3}.$$

$$C = (-5 + 3, -1 + (-4)) = (-2, -5)$$

or

$$C = (-8 + (-3), 3 + 4) = (-11, 7)$$

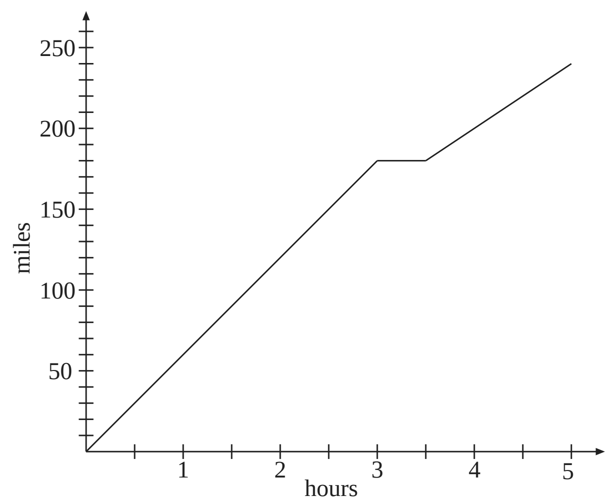
(d) Answers will vary.

$$m(A, B) = \frac{3-3}{-2-5} = \frac{0}{-7} = 0$$

This is a horizontal line. All points have the same y-coordinate, 3. So, possible answers for C are $(-1, 3)$ or $(10, 3)$.

(e) Points A, B, and C are collinear.

2. (a)



(b) Lily travelled 180 miles in 3 hours. Her average speed will be

$$\frac{\text{distance travelled}}{\text{time elapsed}} = \frac{180 \text{ miles}}{3 \text{ hours}} = 60 \text{ miles per hour}$$

(c) Lily travelled 60 miles in $1\frac{1}{2} = \frac{3}{2}$ hours. Her average speed will be

$$\begin{aligned} \frac{\text{distance travelled}}{\text{time elapsed}} &= \frac{60 \text{ miles}}{\frac{3}{2} \text{ hours}} = 60 \cdot \frac{2}{3} \text{ miles per hour} \\ &= 40 \text{ miles per hour} \end{aligned}$$

(d) Lily's entire trip was $180 + 60 = 240$ miles. The time elapsed was $3 + \frac{1}{2} + 1\frac{1}{2} = 5$ hours. Her average speed will be

$$\frac{\text{distance travelled}}{\text{time elapsed}} = \frac{240 \text{ miles}}{5 \text{ hours}} = 48 \text{ miles per hour}$$

3. (a) First find the slope of ℓ:

$$m(\ell) = \frac{5-(-1)}{6-4} = \frac{6}{2} = 3$$

Then find the slope of the line through R and either P or Q:

$$m(R, Q) = \frac{5-2}{6-5} = \frac{3}{1} = 3$$

Since the slopes are the same, R is on ℓ.

(b) For $A = (-2, a)$ to be on ℓ, the slope from A to either P or Q must be 3.

$$\begin{aligned} m(A, Q) &= \frac{a-5}{-2-6} \\ 3 &= \frac{a-5}{-8} \\ -24 &= a - 5 \\ -19 &= a \end{aligned}$$

(c) To find a point-tester equation, you use a point (x, y) and P (or Q) with a slope of 3:

$$\begin{aligned} \frac{y-(-1)}{x-4} &= 3 \\ y + 1 &= 3(x-4) \\ y + 1 &= 3x - 12 \\ y &= 3x - 13 \\ 13 &= 3x - y \end{aligned}$$

4. (a) Write a point-tester equation with a variable point (x, y) and the point $(3, 7)$:

$$\begin{aligned} \frac{y-7}{x-3} &= 2 \\ y - 7 &= 2(x-3) \\ y - 7 &= 2x - 6 \\ y &= 2x + 1 \end{aligned}$$

(b) First find the slope:

$$\frac{-4-0}{2-(-1)} = \frac{-4}{3} = -\frac{4}{3}$$

Then write the point-tester equation with the variable point (x, y) and one of the given points:

$$\begin{aligned} \frac{y-0}{x-(-1)} &= -\frac{4}{3} \\ y - 0 &= -\frac{4}{3}(x+1) \\ y &= -\frac{4}{3}x - \frac{4}{3} \end{aligned}$$

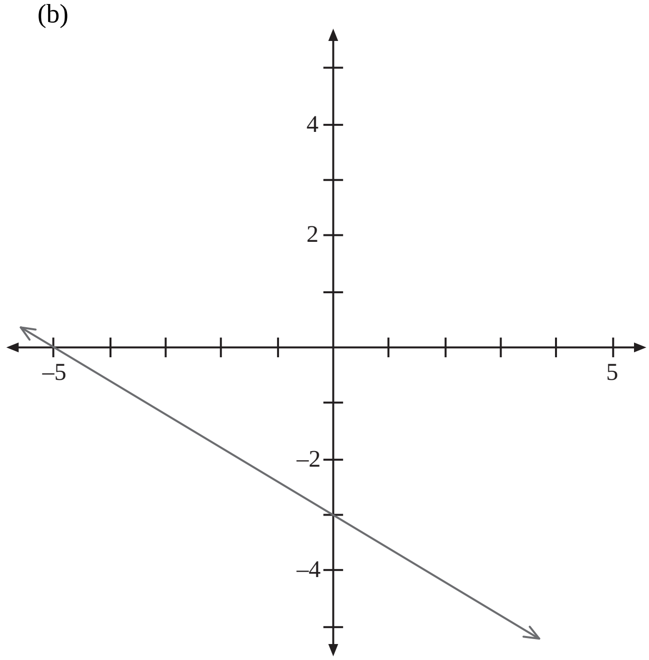
(c) When the slope is 0, the line is horizontal. All points on a horizontal line have the same y-coordinate. The equation is $y = 4$.

(d) When the slope is undefined, the line is a vertical line. All points on a vertical line have the same x-coordinate. The equation is $x = -6$. You cannot write it in the form $y = ax + b$, since there is no y term.

5. (a) Find two points on line ℓ and use these to find the slope. If $x = 0$, $5y = -15 \Longrightarrow y = -3$. If $y = o$, $3x = -15 \Longrightarrow x = -5$. So, two points are $(0, -3)$ and $(-5, 0)$. The slope will be

$$\frac{-3-0}{0-(-5)} = \frac{-3}{5} = -\frac{3}{5}$$

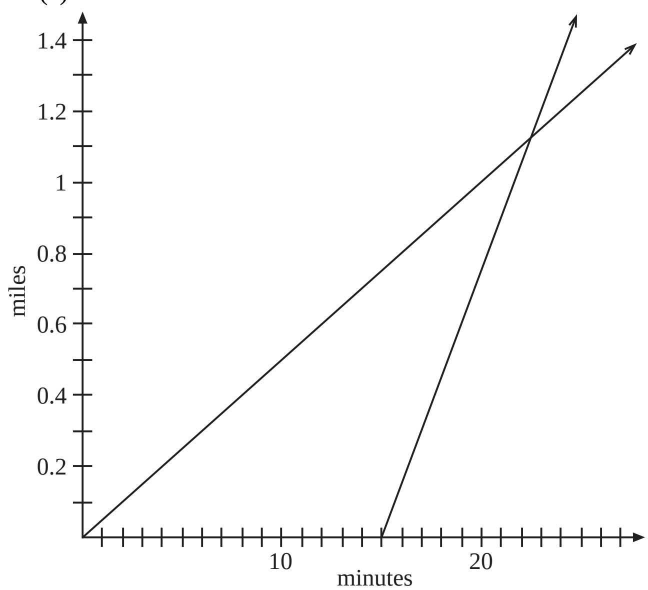
(b)



(c)

$$\begin{aligned} 3x + 5y &= -15 \\ 3x - 3x + 5y &= -15 + 3x \\ 5y &= 3x - 15 \\ \frac{5y}{5} &= \frac{3x-15}{5} \\ y &= \frac{3}{5}x - 3 \end{aligned}$$

6. (a)



(b) If t is the time in hours that Scott's brother is riding, Scott has been walking for 15 minutes longer. 15 minutes is $\frac{15}{60} = \frac{1}{4}$ hour. Scott's time is $t + \frac{1}{4}$ hours. You want to know when Scott's brother will overtake Scott, so set their distances equal:

$$\begin{aligned} 3\left(t + \frac{1}{4}\right) &= 9t \\ 3t + \frac{3}{4} &= 9t \\ \frac{3}{4} &= 6t \\ \frac{1}{6} \cdot \frac{3}{4} &= t \\ \frac{1}{8} &= t \end{aligned}$$

So, Scott's brother will overtake him in $\frac{1}{8}$ hour or $\frac{1}{8} \cdot 60$ minutes $= 7\frac{1}{2}$ minutes.

(c) The distance travelled is the product of the rate and time. So, using Scott's brother's time and rate: $9 \cdot \frac{1}{8} = \frac{9}{8} = 1\frac{1}{8}$. Subtract this from $1\frac{1}{2}$ to find their distance from home will be $\frac{3}{8}$ mile.

7. Answers will vary. You can use a point-tester equation. Choose two different values for the slope. If you let the slope be 2,

$$\begin{aligned} \frac{y-3}{x-(-2)} &= 2 \\ y - 3 &= 2(x+2) \\ y - 3 &= 2x + 4 \\ y &= 2x + 7 \\ -7 &= 2x - y \end{aligned}$$

If you let the slope be -1,

$$\begin{aligned} \frac{y-3}{x-(-2)} &= -1 \\ y - 3 &= -1(x+2) \\ y - 3 &= -x - 2 \\ y &= -x + 1 \\ x + y &= 1 \end{aligned}$$

To show that $(-2, 3)$ solves both equations, substitute -2 for x and 3 for y.

$$\begin{aligned} 2x - y &= -7 \\ 2(-2) - 3 &\stackrel{?}{=} -7 \\ -4 - 3 &\stackrel{?}{=} -7 \\ -7 &= -7 \end{aligned}$$

$$\begin{aligned} x + y &= 1 \\ -2 + 3 &\stackrel{?}{=} 1 \\ 1 &= 1 \end{aligned}$$

8. (a) Solve the first equation for y:

$$\begin{aligned} 4x + y &= 3 \\ 4x - 4x + y &= 3 - 4x \\ y &= 3 - 4x \end{aligned}$$

Now substitute into the second equation:

$$\begin{aligned} 3x - 2y &= 5 \\ 3x - 2(3 - 4x) &= 5 \\ 3x - 6 + 8x &= 5 \\ 11x - 6 &= 5 \\ 11x &= 11 \\ x &= 1 \end{aligned}$$

$y = 3 - 4x = 3 - 4(1) = 3 - 4 = -1$ The solution is $(1, -1)$.

(b) Multiply the second equation by 3 and add to eliminate y:

$$\begin{aligned} x - 3y &= 11 \\ (+)6x + 3y &= 3 \\ 7x &= 14 \\ x &= 2 \end{aligned}$$

To find y, substitute $x = 2$ into either of the original equations:

$$\begin{aligned} x - 3y &= 11 \\ 2 - 3y &= 11 \\ -3y &= 9 \\ y &= -3 \end{aligned}$$

9. Since parallel lines have the same slope, first find the slope of $3x - 2y = 8$ by finding two points on the line. If $x = 0$, $3(0) - 2y = 8 \to -2y = 8 \to y = -4$. If $x = 2$, $3(2) - 2y = 8 \to 6 - 2y = 8 \to -2y = 2 \to y = -1$. Two possible points are $(0, -4)$ and $(2, -1)$. The slope is

$$\frac{-1-(-4)}{2-0} = \frac{3}{2}$$

Now write a point-tester equation:

$$\begin{aligned} \frac{y-(-2)}{x-(-1)} &= \frac{3}{2} \\ y + 2 &= \frac{3}{2}(x+1) \\ 2(y+2) &= 3(x+1) \\ 2y + 4 &= 3x + 3 \\ 4 &= 3x - 2y + 3 \\ 1 &= 3x - 2y \end{aligned}$$

10. (a) Solve the equation: $8x = 24 \to x = 3$. Check a point in each region, say, 0 and 5.

$$8(0) \stackrel{?}{\leq} 24 \to 0 \leq 24$$

$$8(5) \stackrel{?}{\leq} 24 \to 40 \not\leq 24$$

Shade the region that includes 0. Use a closed circle for 3.

$x \le 3$

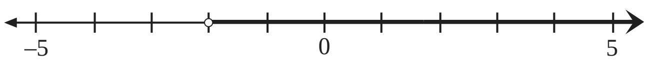
(b) Solve the equation:

$$\begin{aligned} 3(x-1) &= 2x-5 \\ 3x-3 &= 2x-5 \\ x-3 &= -5 \\ x &= -2 \end{aligned}$$

Check a point in each region, say, 0 and -4.

$$\begin{aligned} 3(0-1) &\overset{?}{>} 2(0)-5 \rightarrow -3 > -5 \\ 3(-4-1) &\overset{?}{>} 2(-4)-5 \rightarrow 3(-5) \\ &\overset{?}{>} -8-5 \rightarrow -15 \not> -13 \end{aligned}$$

Shade the region that includes 0. Use an open circle for -2.

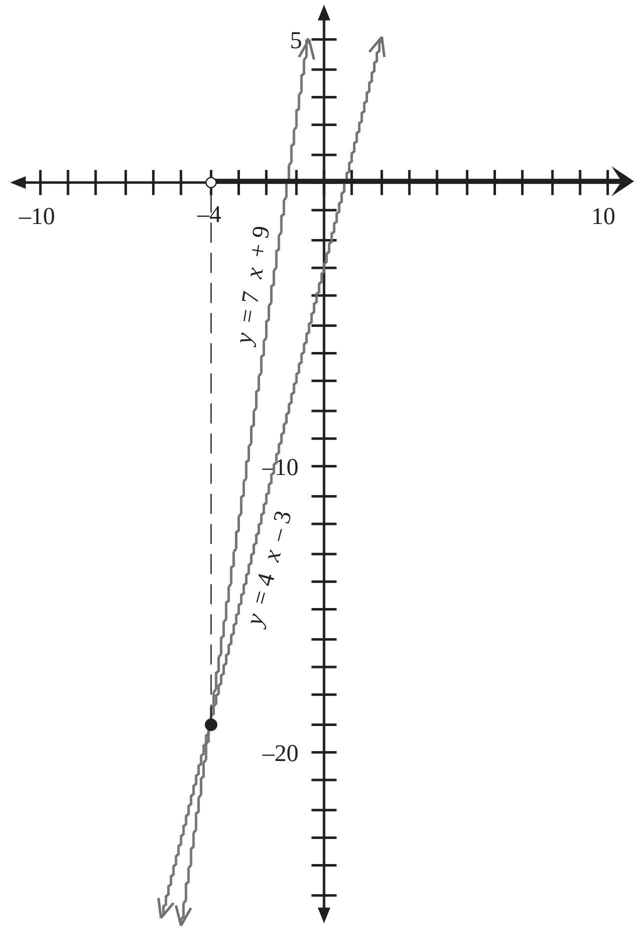


$x > -2$

(c) Graph the two equations $y = 4x - 3$ and $y = 7x + 9$. The solution will be wherever the y-height of the graph of $y = 4x - 3$ is less than the y-height of the graph of $y = 7x + 9$. To find out where the two graphs intersect (and thus the cutoff point of the inequality), solve the equation

$$\begin{aligned} 4x-3 &= 7x+9 \\ -3x &= 12 \\ x &= -4 \end{aligned}$$

The figure below shows the two graphs, with the x-axis serving as the number line solution. Since it is an inequality, use an open circle for -4.



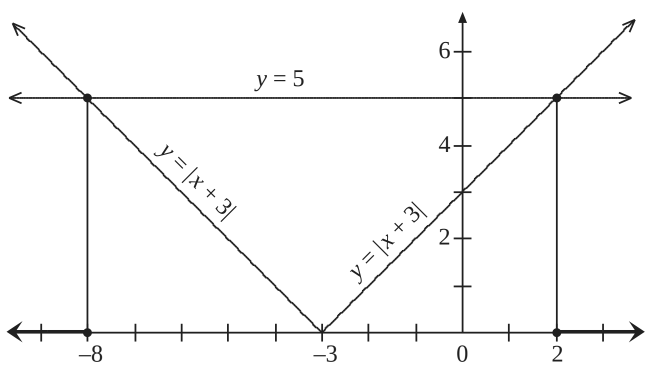
$x > -4$

(d) Graph the two equations $y = |x + 3|$ and $y = 5$ and compare their y-heights. Since the inequality uses $\ge$, the point(s) where the graphs intersect will be included in the solution, so use a closed circle. To find those exact solutions, solve the corresponding equation

$$|x+3| = 5$$

$$\begin{aligned} x+3 &= 5 \quad \text{or} \quad x+3 = -5 \\ x &= 2 \quad \text{or} \quad x = -8 \end{aligned}$$

The figure below shows the two graphs, with the x-axis serving as the number line solution. Use closed circles for -8 and 2.



$x \le -8$ or $x \ge 2$

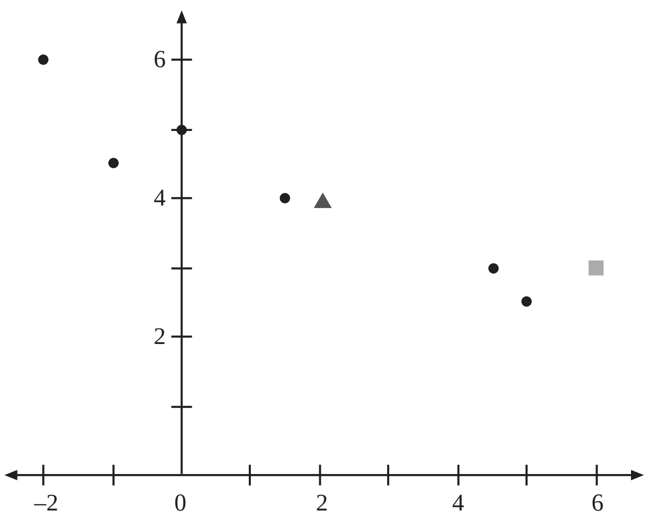
11. (a) You want the average of the x's to be 2:

$$\frac{(-2)+(-1)+0+1.5+4.5+5+a}{7} = 2$$
$$\frac{8+a}{7} = 2$$
$$8+a = 14$$
$$a = 6$$

You want the average of the y's to be 4:

$$\frac{6+4.5+5+4+3+2.5+b}{7} = 4$$
$$\frac{25+b}{7} = 4$$
$$25+b = 28$$
$$b = 3$$

(b)



(c) Negative. The points go down from left to right.

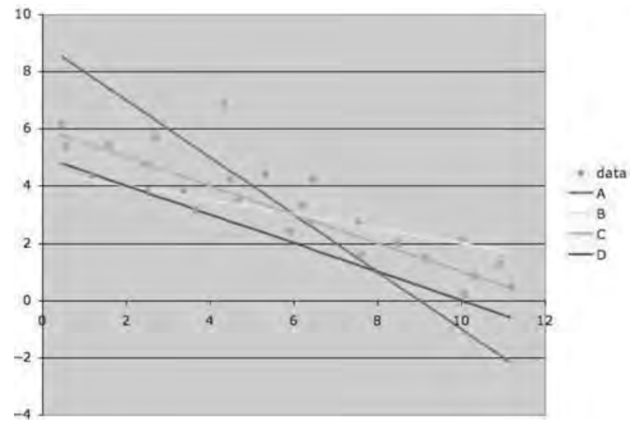
CHAPTER TEST

1. The correct answer is **B**, neither line.
Solve by point-testing. Substitute $x = -3$ and $y = -5$ into each equation. Neither equation becomes true for this point, so the point is on neither line.

2. The correct answer is **A**, Quadrant I.
Sketching both lines suggests the intersection should be in Quadrant I. The idea of overtaking helps here. The graph of $y = 6x - 3$ will "catch up" to $y = 5x + 10$ in Quadrant I.
The actual intersection point is $(13, 75)$.

3. The correct answer is **B**.
The equations in **B** are two different lines with the same slope, so they do not intersect.

4. The correct answer is **C**, $y = -\frac{1}{2}x + 6$.
Only the line $y = -\frac{1}{2}x + 6$ follows the data.



The other lines do not follow the trend.

5. The correct answer is **D**, $x < -2$ or $x > 1.5$.
The line $y = \frac{1}{2}x$ is above the graph of $y = 3 - x^2$ on either side of the intersections. When x is larger than 1.5 or less than -2, the inequality is true. Since the inequality is written as $>$ and not $\geq$, the boundary is not included.

6. The correct answer is **C**, $3a + 7 < 40$.
The value of $3a + 7$ must be less than 37, actually, but it is still true that $3a + 7 < 40$.
The value of $-a$ does not have to be greater than 10; it has to be greater than -10.
The inequality $a \leq 9$ is not necessarily true. For example, a could be 9.5.
The inequality $|a| < 10$ is not necessarily true. For example, a could be -20, in which case $|a|$ would be 20.

7. The system can be solved by elimination, by adding the equations together:

$$\begin{array}{rr} & x + 2y = 17 \\ (+) & 5x - 2y = 16 \\ \hline & 6x \quad = 33 \end{array}$$
$$x = \frac{33}{6} = 5.5$$

Once the value of x is known, substitute to find the value of y:

$$5.5 + 2y = 17$$
$$2y = 11.5$$
$$y = 5.75$$

The system can also be solved by substitution, first solving the first equation as $x = -2y + 17$. The substitution gives the equation

$$5(-2y + 17) - 2y = 16$$

which has the solution $y = 5.75$.
Either way, the solution is $(5.5, 5.75)$.

8. A single bottle of juice costs \$1.25.
Solve by elimination. Double the first order and subtract it from the second:
Twelve bottles of juice and four bags of nuts costs \$23.
Three bottles of juice and four bags of nuts costs \$11.75.

So, nine bottles of juice (the difference) must cost \$11.25. Then, one bottle of juice costs

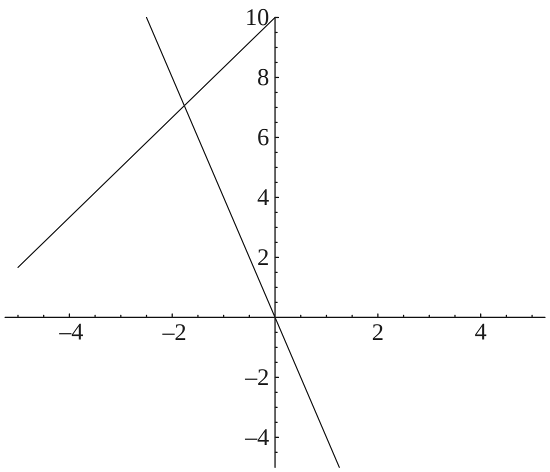
$$\frac{\$11.25}{9} = \$1.25$$

Solving using variables,

$$\begin{array}{rrl} & 12j + 4n &= 23 \\ (-) & 3j + 4n &= 11.75 \\ \hline & 9j &= 11.25 \\ & j &= 1.2 \end{array}$$

It is also possible to solve for n by elimination or by substitution. The value of n is \$2, which can then be used to find j.

9. Here are the graphs:



Find the intersection point by substitution. In the first equation, replace y with $-4x$:

$$5x - 3(-4x) = -30$$

This gives the equation $17x = -30$, which has the solution $x = -\frac{30}{17}$. Then the equation $y = -4x$ gives the value of y:

$$y = -4\left(-\frac{30}{17}\right) = \tfrac{120}{17}$$

The intersection point is thus $\left(-\frac{30}{17}, \frac{120}{17}\right)$.

10. Find the cutoff point by solving the equality

$$\begin{aligned} -3a + 15 &= -3 \\ -3a &= -18 \\ a &= 6 \end{aligned}$$

The cutoff point is an open circle, since the value $a = 6$ does not make the inequality true.

Then, test a convenient point on either side of $a = 6$. Testing $a = 0$ gives

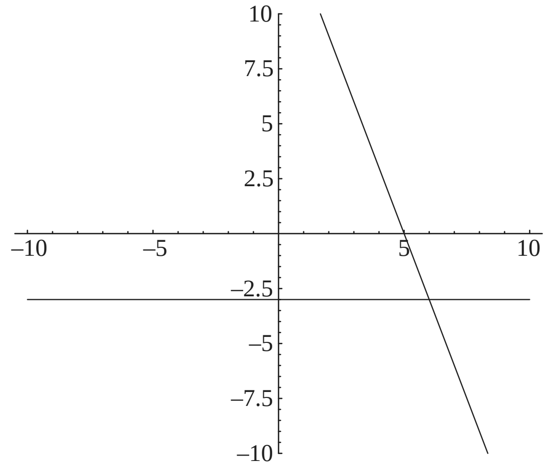
$$15 > -3$$

which is true. Testing $a = 10$ gives

$$-30 + 15 > -3$$

which is not true. So, the inequality must be true when $a < 6$.

The other way to do this problem is to graph each side. Here are the graphs of $y = -3a + 15$ and $y = -3$ (a horizontal line):



According to the graphs, the line $y = -3a + 15$ is higher when a is less than 6.

Either way, the solution graph is

11. Answers will vary. Two different lines intersect if and only if they do not have the same slope, so answers should talk about finding the slope of each line and comparing them.

12. (a) To find the balance point, $(\bar{x}, \bar{y})$, take the mean of the x-coordinates and the mean of the y-coordinates:

$$\bar{x} = \frac{1+2+3+4+5+6+7}{7} = \frac{28}{7} = 4$$
$$\bar{y} = \frac{20+26+35+41+37+52+55}{7} = \frac{266}{7} = 38$$

The balance point is $(4, 38)$.

(b) The equation to test is $y - 38 = 5(x - 4)$.

Input	**Actual**	**Predicted**	**Error**
1	20	23	−3
2	26	28	−2
3	35	33	2
4	41	38	3
5	37	43	−6
6	52	48	4
7	55	53	2

(c) The equation to test is $y - 38 = 6(x - 4)$.

Input	Actual	Predicted	Error
1	20	20	0
2	26	26	0
3	35	32	3
4	41	38	3
5	37	44	−7
6	52	50	2
7	55	56	−1

(d) The sum of the errors from both tables is 0, since both lines go through the balance point. If you take the absolute value of the errors and add *that*, the sum for the slope 5 line is 22, and the sum for the slope 6 line is 16. The slope 6 line appears to be better.

You might also have remembered that the line of best fit is found by minimizing the sum of the *squares* of the errors. You may not know how to find the exact slope that minimizes the sum of the squares, but you can compare the two. The table below shows the squares of the errors for the two lines:

Slope 5 line		Slope 6 line	
Error	$(\text{Error})^2$	Error	$(\text{Error})^2$
−3	9	0	0
−2	4	0	0
2	4	3	9
3	9	3	9
−6	36	−7	49
4	16	2	4
2	4	−1	1
Sum	82	Sum	72

The Slope 6 line also has the lower sum of the squared errors. So either way, the slope-6 line appears to be the better fit.

Challenge Problem

13. There is more than one possible answer, but one is

$$M = (0, 0)$$
$$N = (34, 17)$$
$$P = (7, 35)$$

First, let $M = (0, 0)$. It can be assigned anywhere. Second, suppose $N = (2, 1)$, to give the slope of $\frac{1}{2}$. Then P is somewhere with coordinates $P = (x, 5x)$. The slope between N and P must be $-\frac{2}{3}$:

$$\frac{5x - 1}{x - 2} = -\frac{2}{3}$$

This gives the equation $3(5x - 1) = -2(x - 2)$, which has solution $x = \frac{7}{17}$. So $P = \left(\frac{7}{17}, \frac{35}{17}\right)$ is a solution. Multiplying all coordinates by 17 gives a triangle whose coordinates are all integers.

CUMULATIVE REVIEW

1. (a) −81
(b) −9
(c) 81
(d) −9

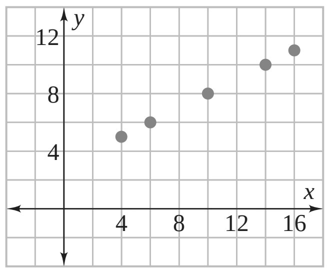
2. (a) Begin by converting the problem into an equation, where n is the starting number. The equation is $3((n + 6) - 2) - 5 = 10$. Solving for n, we see that $n = 1$.
(b) Substitute −5 into the left side of the equation from part (a) to obtain $3((-5 + 6) - 2) - 5 = 3(-1) - 5 = -8$.

3. (a) x is 10 units away from 7.
(b) Rewrite the absolute value equation as two different equations: $x - 7 = 10$ and $x - 7 = -10$. Solving both equations we get $x = 17$ and $x = -3$, respectively.

4. (a) I
(b) III
(c) II

5. Answers may vary. Sample: Runners A and B are competing in a race. Runner B starts out faster, but runner A speeds up and eventually overtakes runner B to win the race.

6. (a)



(b) Answers may vary. Sample: $y = 4$, 7, and 12.

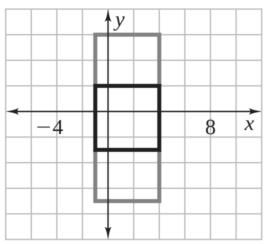
7. 25%

8. (a) The median is represented by the vertical line in the middle of the box; the range is the maximum value (83) minus the minimum value (41).
(b) 41, 49, 64, 69, 83

9. Add 5 to each element in Set I to get Set II.

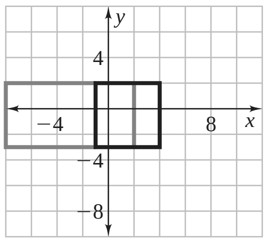
10. The mean and median of Set II are each 5 more than the mean and median of Set I. The ranges of the two sets are the same.

11. (a)



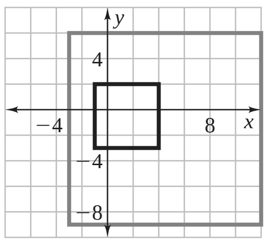
The shape is a rectangle with a width equal to the width of the square, and a height 3 times the height of the square.

(b)



The shape is a rectangle with a height equal to the height of the square, and a width 2 times the width of the square.

(c)



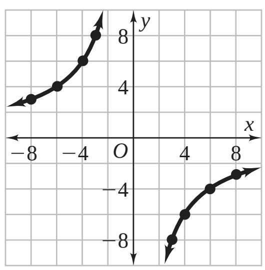
The shape is a square with side lengths 3 times as long as the side length of the original square.

12. To find five points that satisfy $3x - 5y = 45$, choose five values for y, and then solve the resulting equations for x. For example, if $y = 0$, then $3x - 5(0) = 45$, and $3x = 45$. Dividing by 3, you get $x = 15$. Continuing in this way, you find that (20, 3), (10, −3), (30, 9), and (0, −9) also satisfy the equation.

13. This is the easy part. Because (20, 3) satisfies the equation, we know that (20, −1), (20, 0), (20, 1), (20, 2), and (20, 4) do not satisfy the equation.

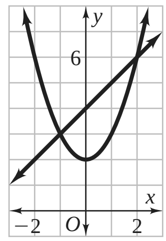
14. Plug in the given values of y, and solve for x. For $y = -5$, we get $\frac{1}{4}(-5) + x = 3$.
Solving for x, we get $x = 4\frac{1}{4} = h$. Continuing in this way, you find $k = 6\frac{1}{4}$ and $m = 6\frac{3}{4}$.

15. (a) Choose six values for y, and solve the resulting equations for x. For example, if $y = 2$, then $x(2) = -24$. Solving for x, you get $x = -12$. In a similar way, you find that (−24, 1), (24, −1), (12, −2), (8, −3), and (4, −6) are on the graph.
(b) The graph passes through quadrants II and IV.
(c) Answers may vary. Sample:



16. $x^3, x, \frac{1}{x}, |x|, x^2$

17.



(−1, 3) and (2, 6)

18. (a) $y = (x-3)^2$
(b) $y = (x+2)^2$
(c) $y + 2 = (x-3)^2$

19. Divide the distance traveled by time, to get $\frac{145 \text{ mi}}{2.5 \text{ hr}} = 58\frac{\text{mi}}{\text{hr}}$.

20. Divide the change in y-values by the change in x-values to get $\frac{-2-6}{5-(-3)} = \frac{-8}{8} = -1$.

21. (−4, 3) is on the line because the slope from (−3, 1) to (−4, 3), and the slope from (−4, 3) to (1, −7) are equal. The other two points are not on the line because there are three different slopes between the points in each case.

22. Use the point-slope form: $\frac{y-(-11)}{x-3} = -4$. Solving for y, you get $y = -4x + 1$.

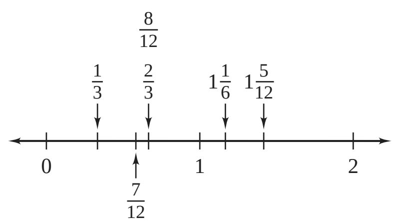
23. Again use the slope-intercept form: $\frac{y-(-11)}{x-12} = -\frac{1}{3}$. Solve for y to get $y = -\frac{1}{3}x - 7$.

24. (a) line; slope $= \frac{1}{5}$; (0, 5), (−25, 0)
(b) not a line
(c) line; slope = 6; (0, 0)

25. Substitute $b = 3$ into $2a - b = -5$ to get $2a - 3 = -5$. Solving for a you get $a = -1$.

26. Let x be the price of soup, and y be the price of salad. Then you can form two equations: $4x + y = 8.5$ and $4y = 10$. Solving the second equation for y, we get $y = 2.5$. Substituting this into the first equation, you get $4x + 2.5 = 8.5$. Solving for x, you get $x = 1.5$. So, the price of one soup is \$1.50, and one salad is \$2.50. Therefore, the price of four soups is \$6.00.

27. (a)



(b) Yes, $\frac{2}{3}$ and $\frac{8}{12}$ are equivalent.
(c) $\frac{1}{3}$
(d) $1\frac{5}{12}$

28. (a) Add 5 and then divide by 2.
(b) Subtract 7 and then divide by −4.
(c) Multiply by 6 and then subtract 3.

29. (a) Subtract 94 from each side.
(b) Add $6y$ to each side.
(c) Subtract $5x$ from each side and then divide each side by −6.
(d) Add $6y$ to each side and then divide each side by 5.

30. $m = 4$

31. $p = 1$

32. $q = 15$

33. Let p be last year's price of a television. Then the price of televisions this year is $p - 100$, and the equation relating the two prices is $p - 100 = 0.80p$. Solving for p, you get $p = \$500$.

34. $x = 7$, $x = -8$, $x = 4$