

### Normal Model Drill

Suppose the results of an exam, taken by a class with a large number of students, follow a normal model with mean 60 and standard deviation 8.

Answer the following questions regarding the exam:

- What proportion of the class had scores between 52 and 68?
- What proportion of the class had scores between 36 and 84?
- What proportion of the class had scores below 36?
- What proportion of the class had scores above 76?
- What score do you need to ensure that you are in the top 2.5% of the class?
- What score do you need to ensure that you are not in the bottom 2.5% of the class?
- What proportion of the class had scores between 50 and 70?
- What proportion of the class had scores between 70 and 80?
- Suppose you scored a 72 on the test. How many percent of the students had better scores? How many percent had worse scores?
- Suppose you scored a 25 on the test. How many percent of the students had better scores? How many percent had worse scores?
- What score do you need to ensure that you are in the top 10% of the class?
- What score do you need to ensure that you are in the top 20% of the class?

**Instructions:** Complete the questions on the following pages.

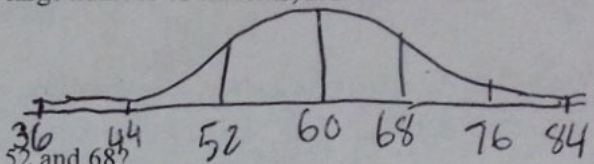
- (1) Suppose a data set is represented by a normal distribution with a mean of 125 and a standard deviation of 7.
- (a) What data value is 2 standard deviations above the mean? \_\_\_\_\_
  - (b) What data value is 3 standard deviations below the mean? \_\_\_\_\_
  - (c) What data value is 1.5 standard deviations below the mean? \_\_\_\_\_
  - (d) What data value is 2.5 standard deviations above the mean? \_\_\_\_\_
  - (e) What data value is  $\frac{1}{5}$  of a standard deviation above the mean? \_\_\_\_\_
- (2) What are the  $z$ -scores for the data values in parts (a)-(e) of the previous question?
- (a) \_\_\_\_\_
  - (b) \_\_\_\_\_
  - (c) \_\_\_\_\_
  - (d) \_\_\_\_\_
  - (e) \_\_\_\_\_
- (3) How do the  $z$ -scores from problem (2) correspond to the standard deviations given in problem (1)? What does this tell you about the relationship between  $z$ -scores for a normal distribution and standard deviation?
- (4) Suppose a normal distribution has mean 10 and standard deviation 2. Find the  $z$ -scores of the measurements 9, 10, 11, 14, and 17.
- (5) In a normally distributed data set, find the value of the standard deviation if the following additional information is given.
- (a) The mean is 226.2 and the  $z$ -score for a data value of 230 is 0.2.  $\sigma =$  \_\_\_\_\_
  - (b) The mean is 9.8 and a  $z$ -score for the data value of 10.3 is 2.  $\sigma =$  \_\_\_\_\_
  - (c) The mean is 14.6 and the  $z$ -score for a data value of 5 is  $-0.3$ .  $\sigma =$  \_\_\_\_\_
  - (d) The mean is 577 and the  $z$ -score for a data value of 533 is  $-0.5$ .  $\sigma =$  \_\_\_\_\_



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- What proportion of the class had scores between 52 and 68?  
68%
- What proportion of the class had scores between 36 and 84?  
99.7%      99.7%
- What proportion of the class had scores below 36?  
.15%
- What proportion of the class had scores above 76?  
2.5%
- What score do you need to ensure that you are in the top 2.5% of the class?  
76
- What score do you need to ensure that you are not in the bottom 2.5% of the class?  
44

• What proportion of the class had scores between 50 and 70?  
 $X=50 \rightarrow Z = \frac{50-60}{8} = -1.25$   
 $Z = -1.25$   
  
 $X=70 \rightarrow Z = \frac{70-60}{8} = +1.25$   
 $Z = +1.25$   
 .89435  
 -.10565  
 about .7887  $\rightarrow$  79%

• What proportion of the class had scores between 70 and 80?  
 $X=70 \rightarrow Z = 1.25$  and  $Z = 2.5$   
 .89435      .99379  
 -.89435  
 .09944  
 about 10%

• Suppose you scored a 72 on the test. How many percent of the students had better scores? How many percent had worse scores?  
 $X=72 \rightarrow Z = \frac{72-60}{8} = 1.5$   
 .93319 area  
 93.3% had worse scores  
 Scores 6.7% had better

• Suppose you scored a 25 on the test. How many percent of the students had better scores? How many percent had worse scores?  
 $X=25 \rightarrow Z = \frac{25-60}{8} = -4.375$   
 normalcdf(-1000000, -4.375, 1, 0)  
 Area = .000006  
 MORE THAN 40% BELOW THE MEAN!!!  
 Everybody did better  
 Nobody did worse.

• What score do you need to ensure that you are in the top 10% of the class?  
  
 10% area  
 $1 - .1000 = .9000$   
 area to the left  
 $Z = 1.28$   
 $X = ? + 1.28 = \frac{X-60}{8}$   
 $X = 70.24$

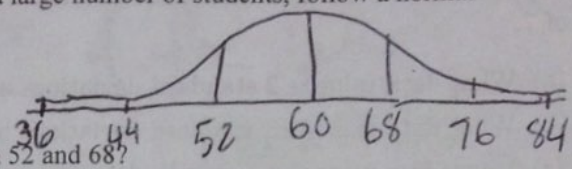
• What score do you need to ensure that you are in the top 20% of the class?  
  
 20% area  
 $1 - .20 = .8000$   
 area  
 $Z = .84$   
 $.84 = \frac{X-60}{8}$   
 $6.72 = X - 60$   
 $+60.00$   
 $+60$   
 $66.72 = X$



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$X=50 \rightarrow Z = \frac{50-60}{8} = -1.25$   
 $Z = -1.25$   
  
 $X=70 \rightarrow Z = \frac{70-60}{8} = +1.25$   
 $Z = +1.25$   
 .89435  
 .10565 about  
 .89435  
 .99379  
 .09944 about 10%

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 How many percent had worse scores?  
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 93.3% had worse scores  
 6.7% had better scores  
 normalcdf(-100000, -4.375, 1, 0)  
 $X=25 \rightarrow Z = \frac{25-60}{8} = -4.375$   
 Area = .000006  
 MORE THAN 40% BELOW THE MEAN!!!  
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 10% area  
 .1000  
 $1 - .1000 = .9000$   
 area to the left  
 $Z = 1.28$   
 $X = ? + 1.28 = \frac{X-60}{8}$   
 $X = 70.24$

What score do you need to ensure that you are in the top 20% of the class?  
  
 20% area  
 $1 - .20 = .8000$   
 area  
 $Z = .84$   
 $.84 = \frac{X-60}{8}$   
 $6.72 = X - 60$   
 $+60.00$   
 $+60$   
 $66.72 = X$